Risk Measures

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AVaR for general distributions

Proposition

AVaR is a coherent risk measure.

Proposition

Given $Z \in L^1$.

1. If $0 < \alpha \leq \beta < 1$, then $AVaR_\beta(Z) \leq AVaR_\alpha(Z)$.
2. $\lim_{\alpha \uparrow 1} AVaR_\alpha(Z) = \mathbb{E}[Z] = \inf_u u + \mathbb{E}\left[[Z-u]^+\right]$.
3. $\lim_{\alpha \downarrow 0} AVaR_\alpha(Z) = VaR_0 E(Z) = \sup Z := \sup\{t : \mathbb{P}[Z > t] > 0\}$.
4. The function $\alpha \to AVaR_\alpha(Z)$ is left continuous.

The level α can be regarded as being a risk tolerance level.

AVaR for general distributions

Remark

For $Z \in L^1$ and $0 < \alpha < 1$ we have

$$AVaR_\alpha(Z) = \left(1 - \frac{\mathbb{P}[Z > VaR_\alpha(Z)]}{\alpha}\right) VaR_\alpha(A) + \frac{\mathbb{P}[Z > VaR_\alpha(Z)]}{\alpha} \mathbb{E}[Z|Z > VaR_\alpha(Z)]$$

and

$$AVaR_\alpha(Z) = \left(1 - \frac{\mathbb{P}[Z \geq VaR_\alpha(Z)]}{\alpha}\right) VaR_\alpha(A) + \frac{\mathbb{P}[Z \geq VaR_\alpha(Z)]}{\alpha} \mathbb{E}[Z|Z \geq VaR_\alpha(Z)]$$

Thus,

$$\mathbb{E}[Z|Z \geq VaR_\alpha(Z)] \leq AVaR_\alpha(Z) \leq \mathbb{E}[Z|Z > VaR_\alpha(Z)]$$