# Two-stage Stochastic Programming Problems

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# The Interchangeability Property

## Two Stage Problem

$$
\text{First Stage}\begin{cases} \min_x & f(x) + \mathbb{E}[Q(x,\xi)] \\ \text{s. t.} & x \in X \end{cases}
$$

$$
\text{Second Stage}\begin{cases} Q(x,\xi) := & \min_y \quad g(y,\xi) \\ & \text{s. t.} \quad y \in \mathcal{G}(x,\xi) \end{cases}
$$

- It is crucial $Q(x,\cdot)$ to be measurable, for all $x$.
- Easy case: $\xi$ discrete random vector (since $Q(x,\cdot)$ is a discrete random variable).
- Example: Consider
  - $\Omega = [0,1]$ with the Lebesgue measure.
  - $\xi(\omega) = \omega$ (uniform distribution)
  - $\mathcal{G}(x,\xi) = [0,1]$
  - $g(y,\xi) = \operatorname{sign}\big((\xi - y\chi_N(y))^2\big)$, where $N \subset [0,1]$ is a nonmensurable set.

# The Interchangeability Property

## One Stage Problem

$$\begin{array}{cl} \min_{x,y_\xi} & f(x) + \mathbb{E}[g(y_\xi,\xi)] \\ \text{s. t.} & x \in X \\ & y_\xi \in \mathcal{G}(x,\xi), \quad \text{a.e. } [\xi] \end{array}$$

- $g(y_\xi,\xi)$ needs to be measurable, for all $x$.
- Measurability of $y_\xi$ is not required.

# The Interchangeability Property

## Proposition

Assuming that $Q(x, \xi)$, $g(y_\xi, \xi)$ and the minimizers $\bar{y}_\xi$ are measurable, then the one stage and two stage problems are equivalent.

# The Interchangeability Property

## Definition

A function $f : \mathbb{R}^n \times \mathbb{R}^q \to \overline{\mathbb{R}}$ is a Carathéodory function if

1. $f(x, \cdot)$ is measurable for all $x$.
2. $f(\cdot, \xi)$ is continuous for a. e. $[\xi]$.

## Theorem

Given the problem

$$Q(x, \xi) := \begin{array}{ll} \min_y & g(y, \xi) \\ \text{s. t.} & c(y, \xi) \leq x \end{array}$$

If $g(y, \xi)$ and all components of $c(y, \xi)$ are Carathéodory functions, then $Q(x, \xi)$ is measurable, for all $x$.