ALGORITHMS FOR TWO-STAGE SP: A PRIMER ON NONSMOOTH OPTIMIZATION

Claudia Sagastizábal

BAS Lecture 9, April 7, 2016, IMPA

SVAN 2016

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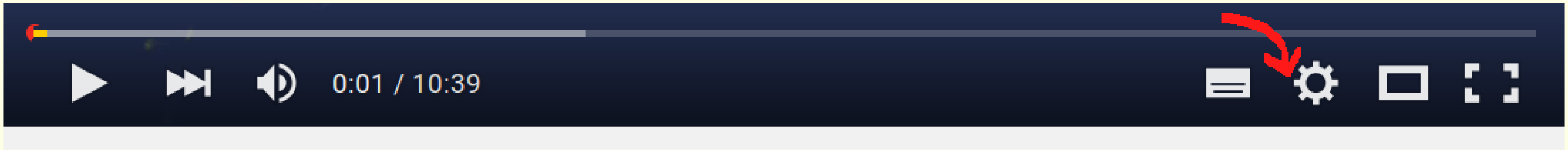

for best viewing

Two-Stage LP with RCR[a], $\Omega = \{\omega^1, \dots, \omega^S\}$

$$\min_{\mathbf{x} \in \mathbf{X}} \mathbf{c}^\top \mathbf{x} + \phi(\mathbf{x}) \qquad \text{for} \qquad \mathbf{X} := \{\mathbf{x} \geq \mathbf{0} : \mathbf{A}\mathbf{x} = \mathbf{b}\},$$

$$\text{where } \phi(x) = \mathbb{E}\left[Q(x, \xi)\right] = \sum_{s=1}^{S} p_s Q(x, \xi^s) \qquad \text{and}$$

$$Q(x, \xi^s) = \begin{cases} \min & q^{s\top} y \\ \text{s.t.} & W^s y = h^s - T^s x \\ & y \geq 0 \end{cases} = \begin{cases} \max & \pi^\top (h^s - T^s x) \\ \text{s.t.} & W^{s\top} \pi \leq q^s \end{cases}$$

$$\partial\phi(x^{\mathbf{k}}) = -\sum_{s=1}^{S} p_s T^{s\top} \arg\max\left\{\pi^\top (h^s - T^s x^{\mathbf{k}}) : \pi \in \Pi(q^s)\right\}$$

[a] next lecture: without Relative Complete Recourse (infeasibility!)

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Primal solution $y^{s,k}$ Dual solution $\pi^{s,k}$

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Evaluating $\phi(x^k) = \sum_{s=1}^{S} p_s \pi^{s,k\,\top}(h^s - T^s x^k)$

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the linearization

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\begin{aligned}
&\geq \quad \phi(x^k) + \gamma^{k\top}(x - x^k) \\
&= \quad \sum_{s=1}^{S} p_s \pi^{s,k\top}(h^s - T^s x^k) - \sum_{s=1}^{S} p_s \pi^{s,k\top} T^s (x - x^k) \\
&= \quad \sum_{s=1}^{S} p_s \pi^{s,k\top}(h^s - T^s x)
\end{aligned}
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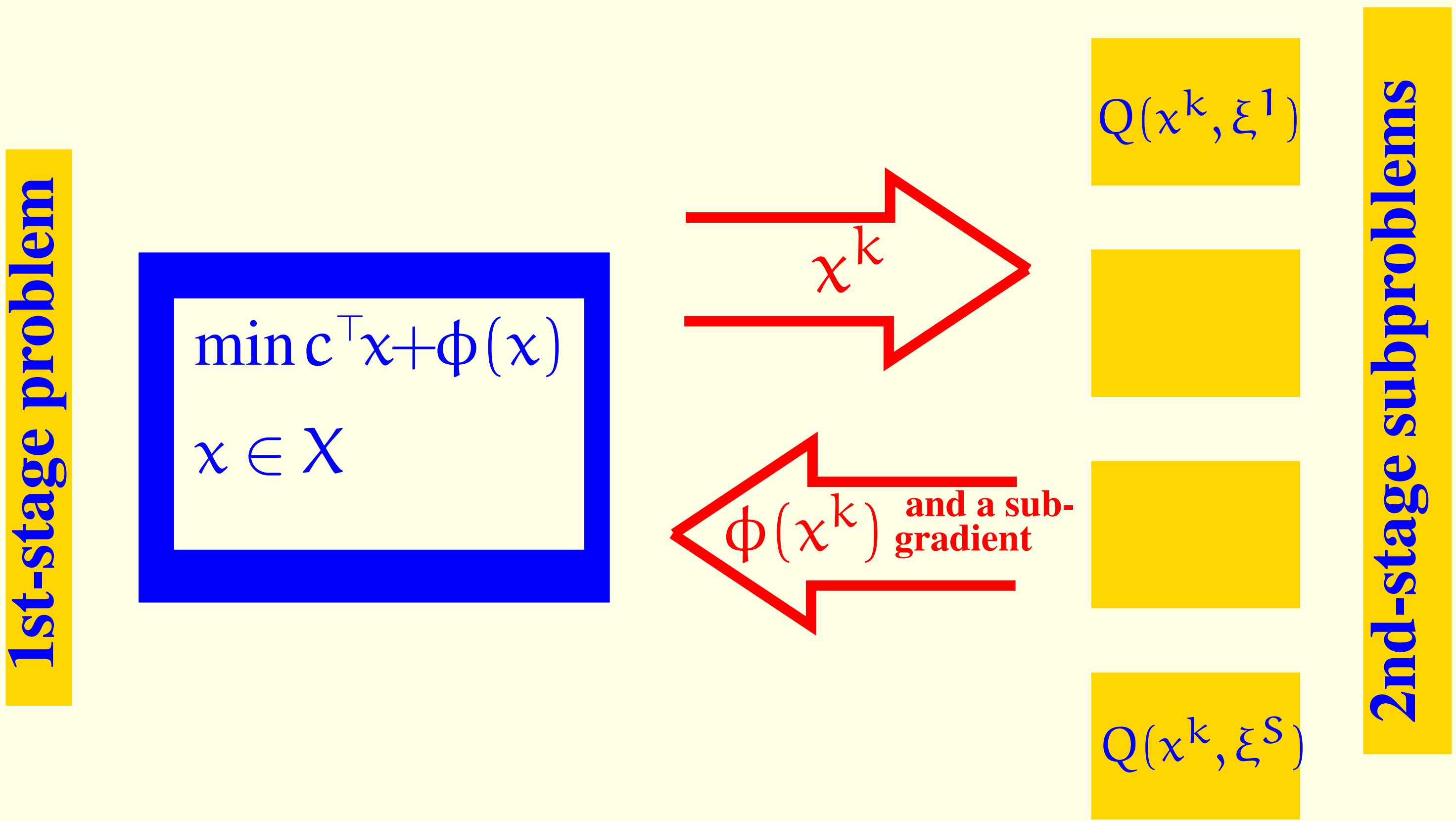
\end{aligned}$$

Evaluating ϕ at x^k



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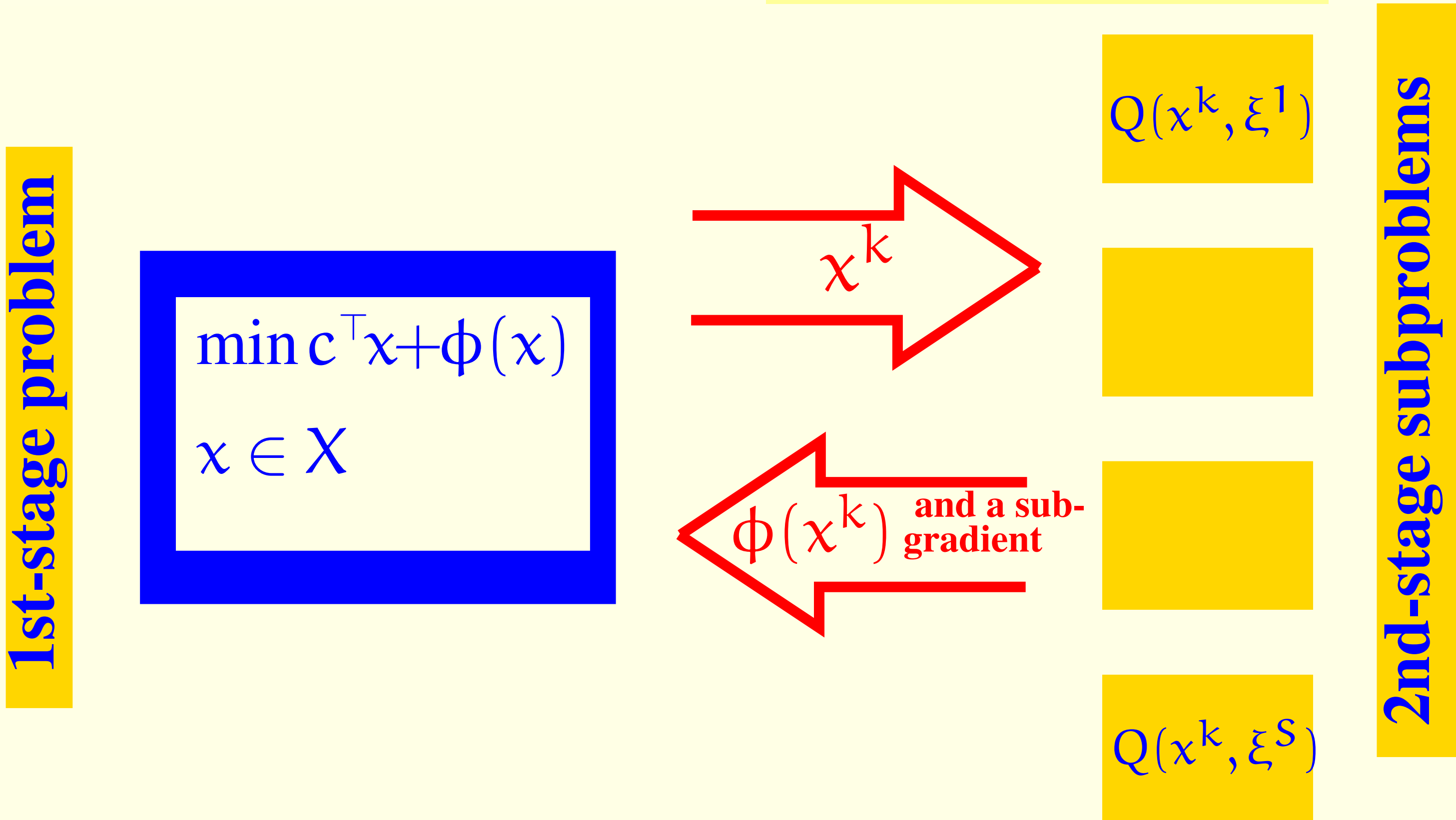

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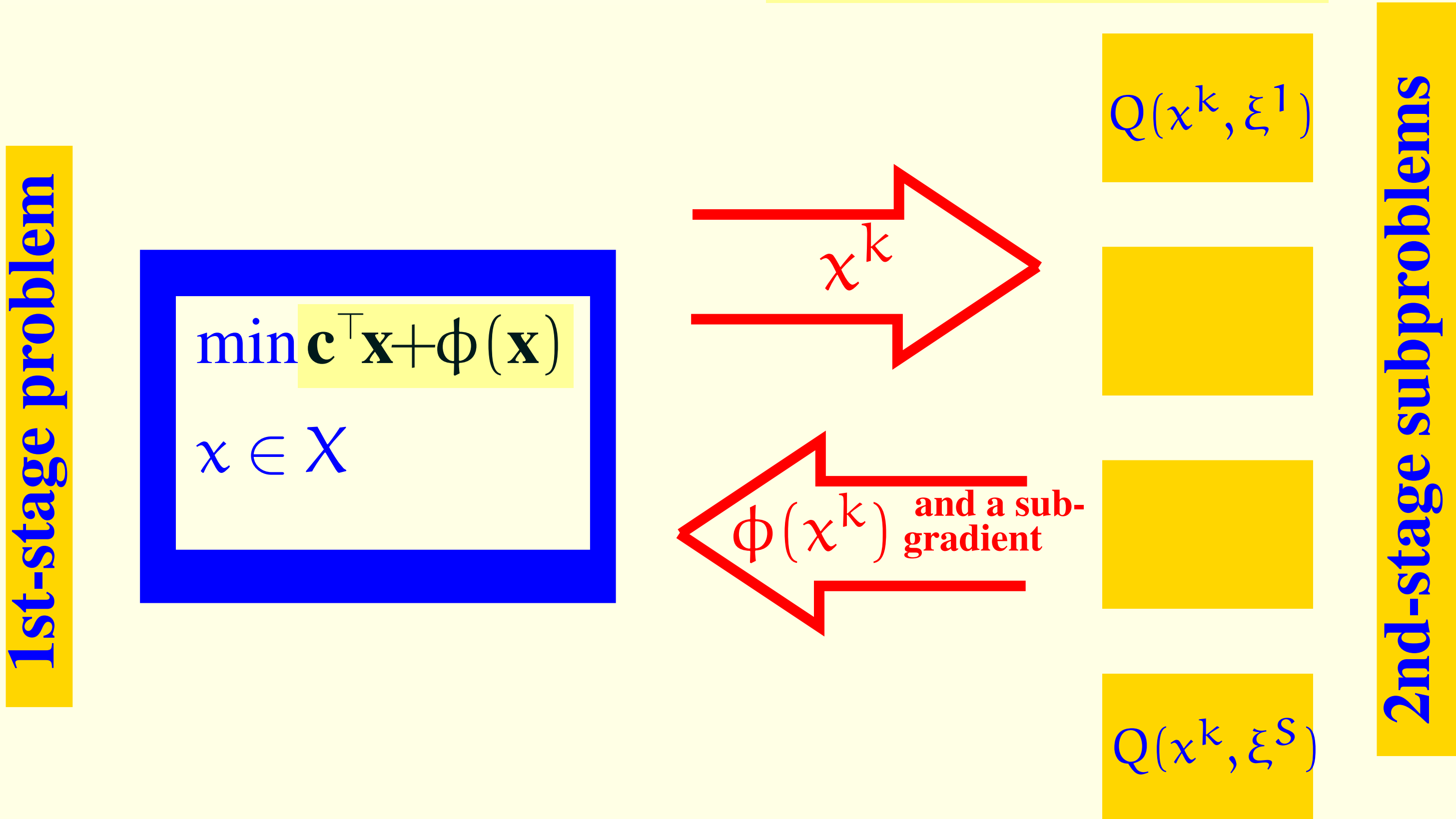


$\min_{x \in X} f(x)$ convex nonsmooth knowing f(x) and $g(x) \in \partial f(x)$ (one)

Computational NSO: what does it mean?

For the unconstrained[a] problem

$$\boxed{\min f(x)},$$

where f is convex but not differentiable at some points

[a] $X = \mathbb{R}^n$ today

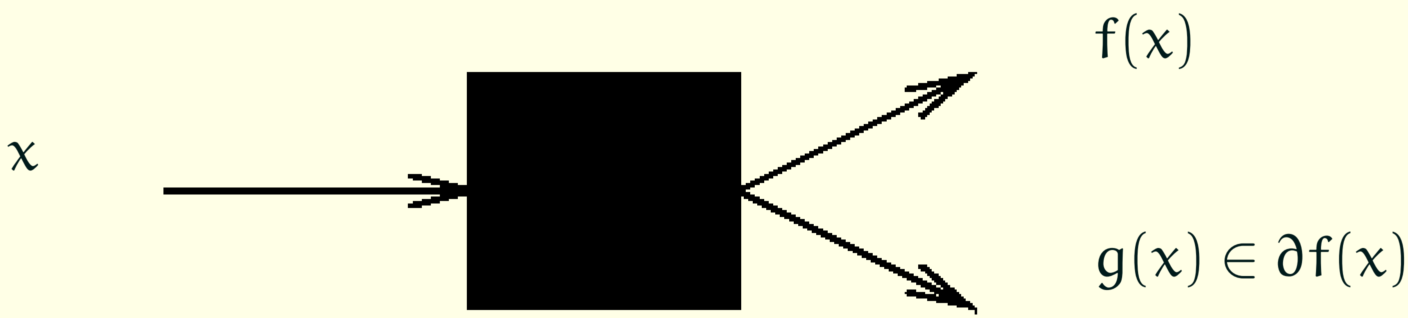
Computational NSO: what does it mean?

For the unconstrained problem

$$\boxed{\min f(x)},$$

where f is convex but not differentiable at some points,

we shall define **algorithms** based on information provided by an oracle or "black box"



What do we mean by an algorithm?

An example

source `http://comofas.com/`

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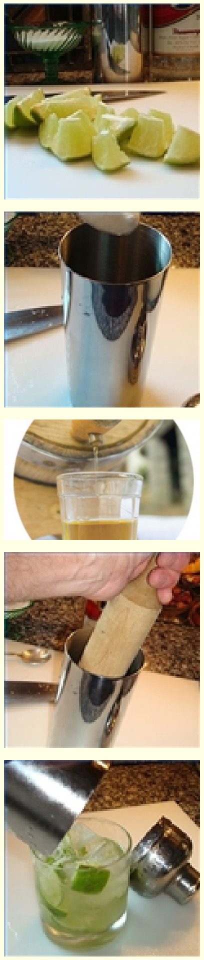

repeat until . . . ??

What do we mean by an algorithm?

An algorithm

is a sequence of steps

that are repeated

until satisfaction

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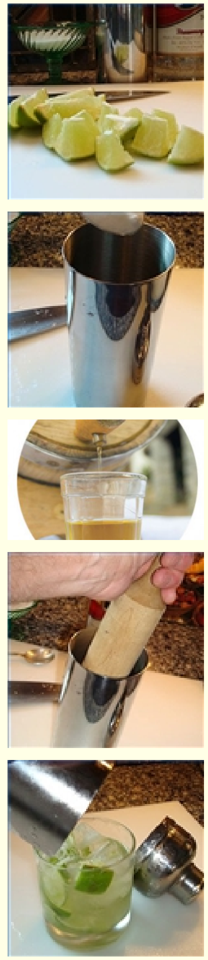


An algorithm

is a sequence of steps

that are repeated

until satisfaction

of a stopping test

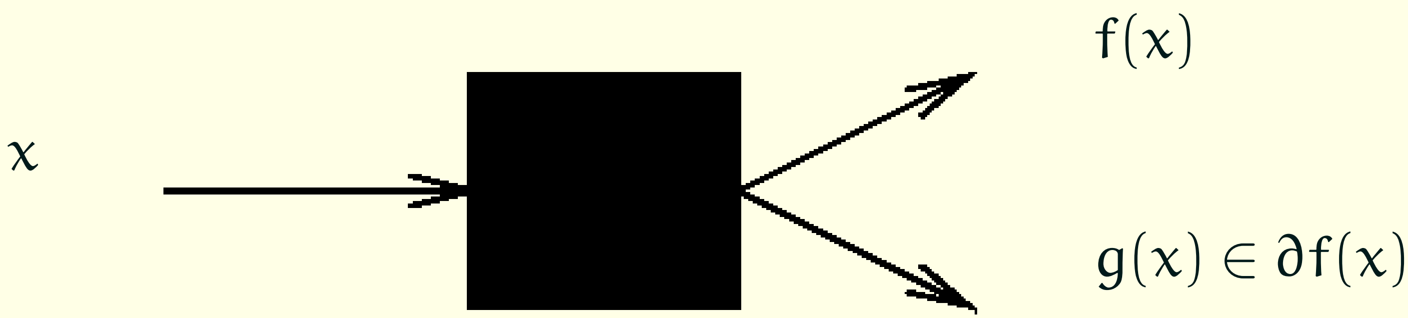
Back to Computational NSO

For the unconstrained problem

$$\min f(x),$$

where f is convex but not differentiable at some points,

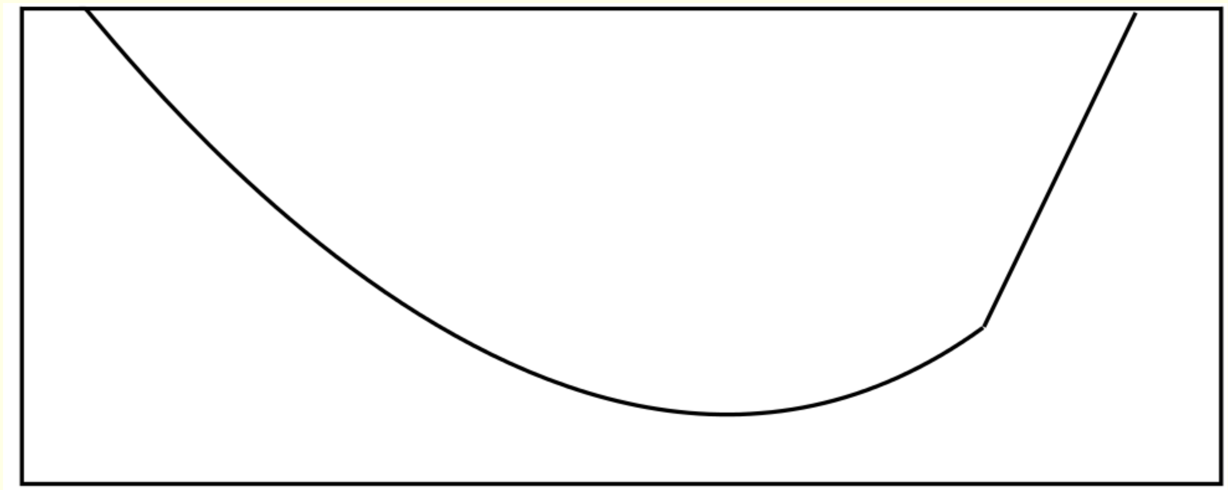
we look for algorithms based on information provided by an oracle or “black box”



endowed with reliable **stopping tests**

What can be done with the oracle information?

An example of a convex nonsmooth function



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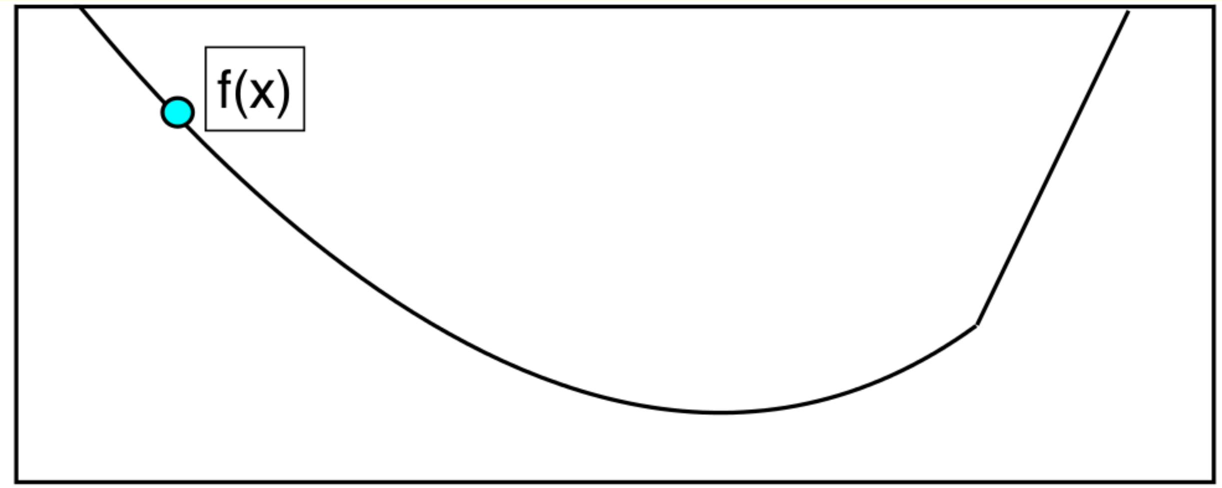

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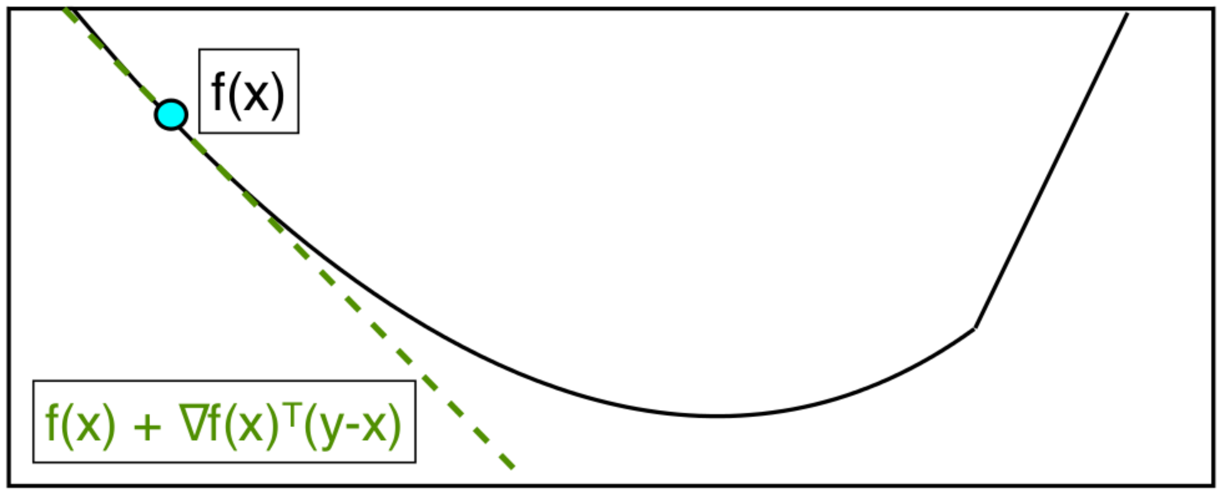

$$
\begin{aligned}
\partial f(x) &= \{\nabla f(x)\} \\
&= \{\text{slopes of linearizations supporting } f\text{, tangent at } x\}
\end{aligned}
$$

What can be done with the oracle information?

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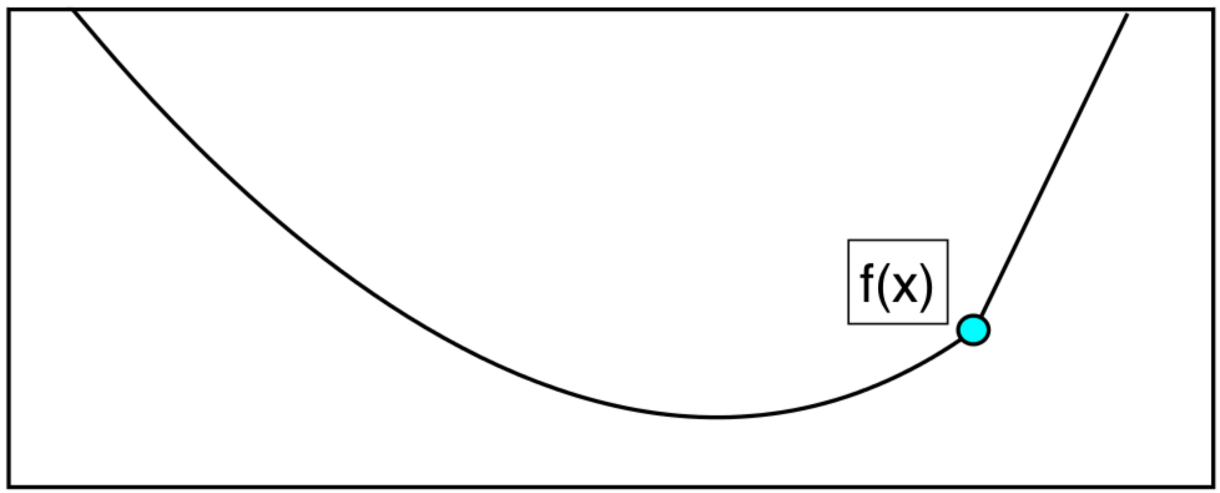

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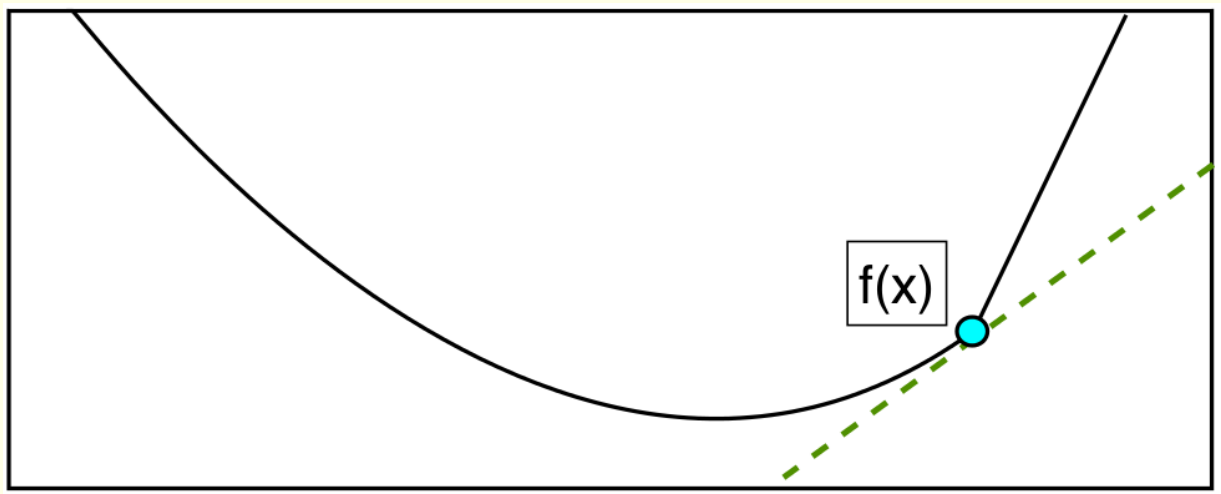

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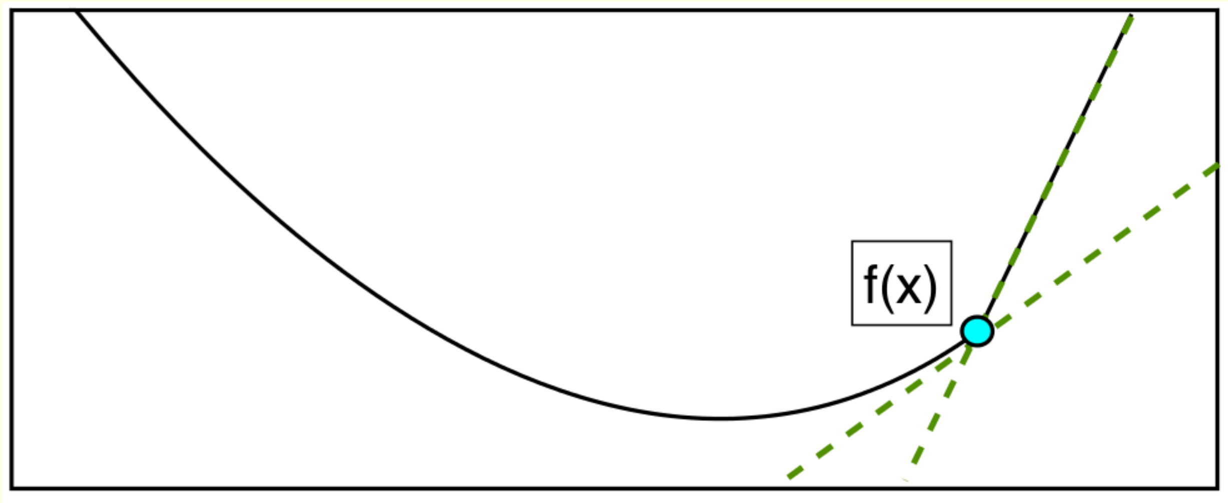

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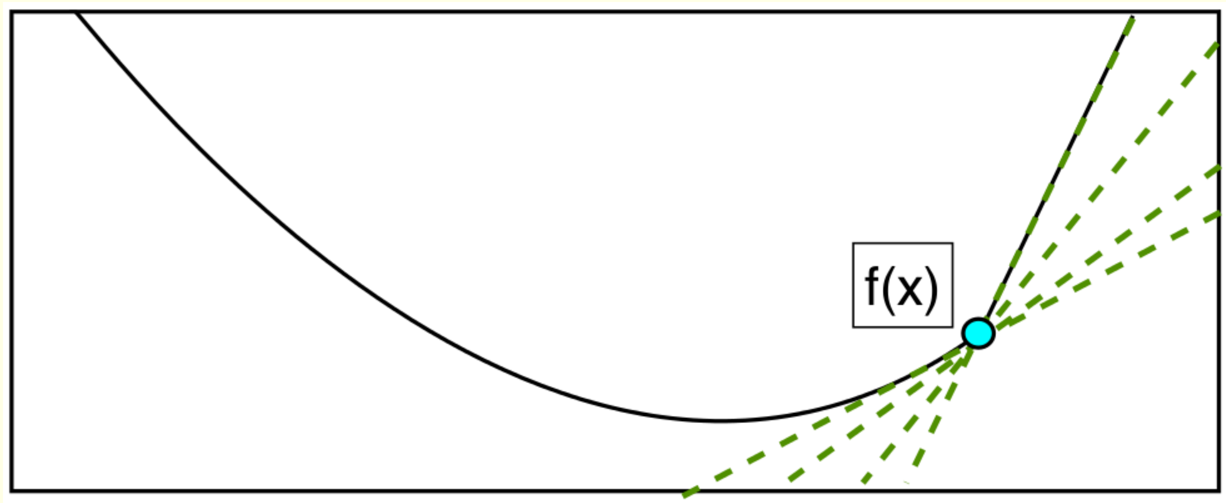

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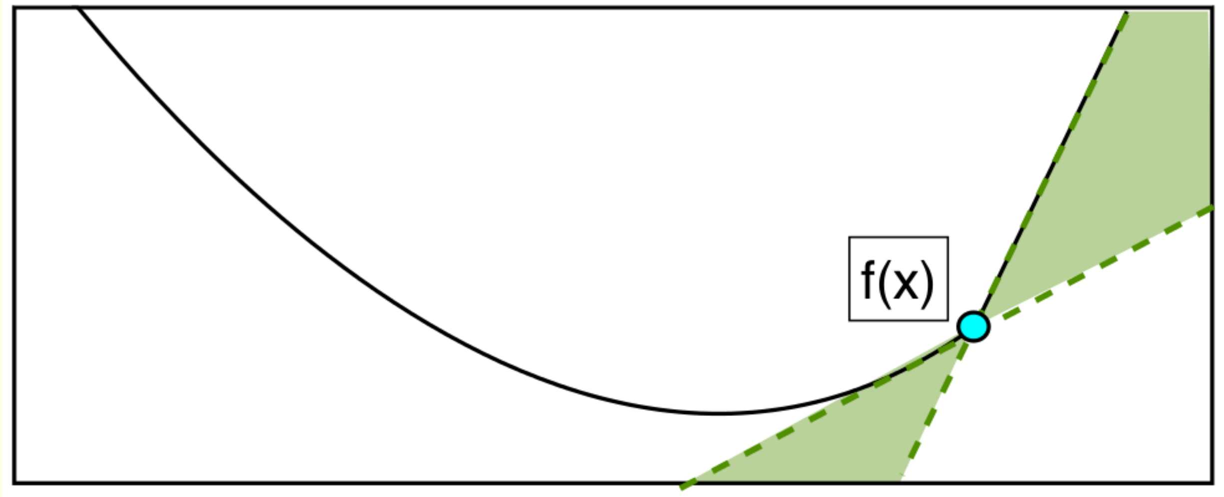

$$\partial f(x) \quad = \quad \{g \in \mathbb{R}^n : f(y) \geq f(x) + g^\top (y - x) \text{ for all } y\}$$

f $\qquad$ x

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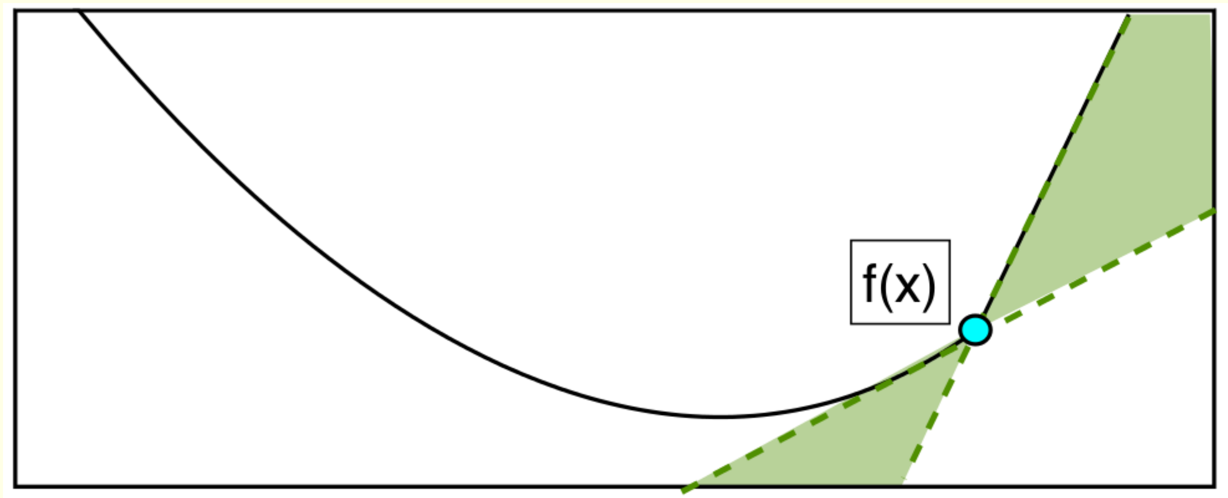

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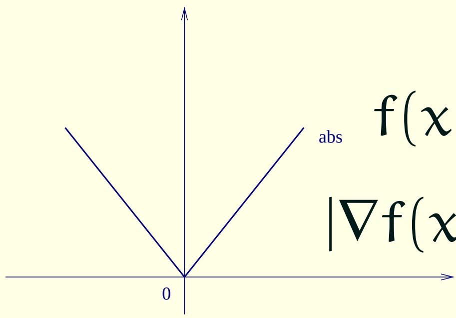
Why special NSO methods?

Smooth optimization methods **do not work**



$$f(x) \quad = |x|$$

$$|\nabla f(x^k)| \quad = 1,\ \forall x \neq 0 \qquad \partial f(0) = [-1, 1]$$

Smooth stopping test **fails**: $|\nabla f(x^k)| \leq$TOL $\qquad (\leftrightarrow |g(x^k)| \leq$TOL$)$

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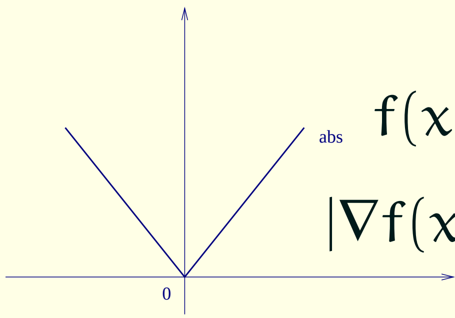

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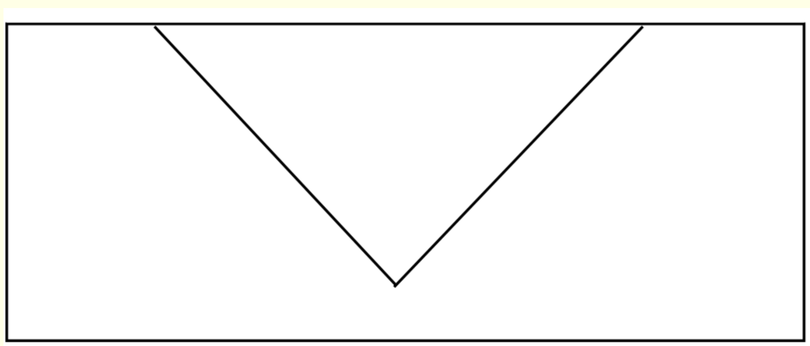

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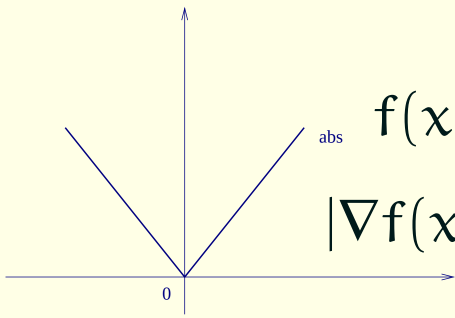

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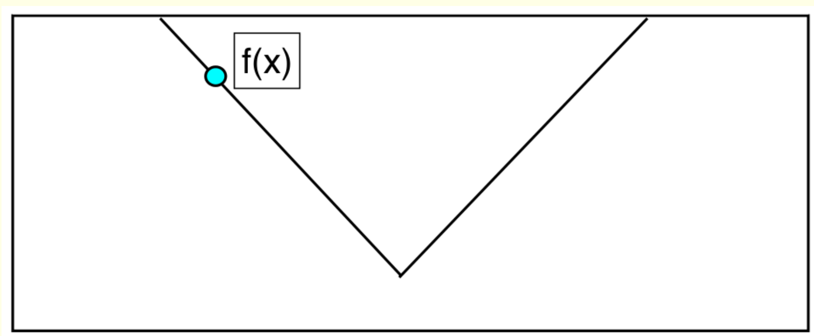

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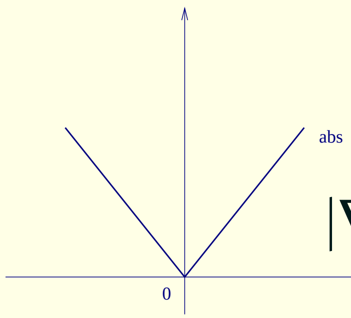

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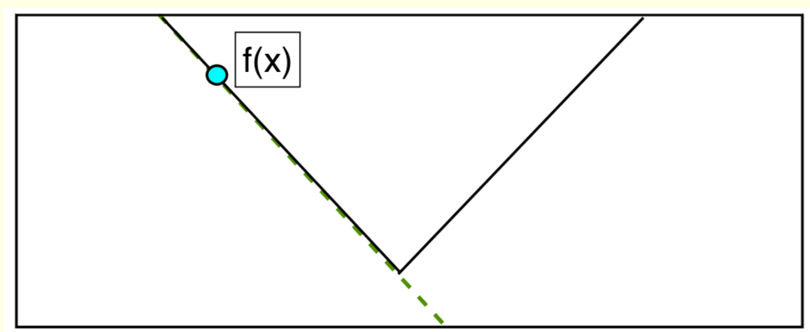

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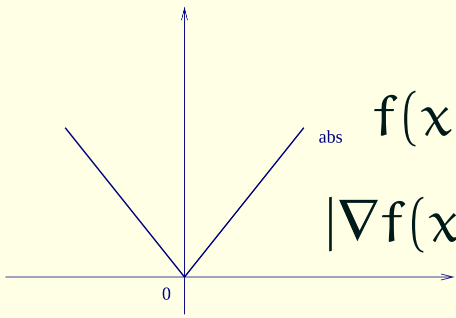

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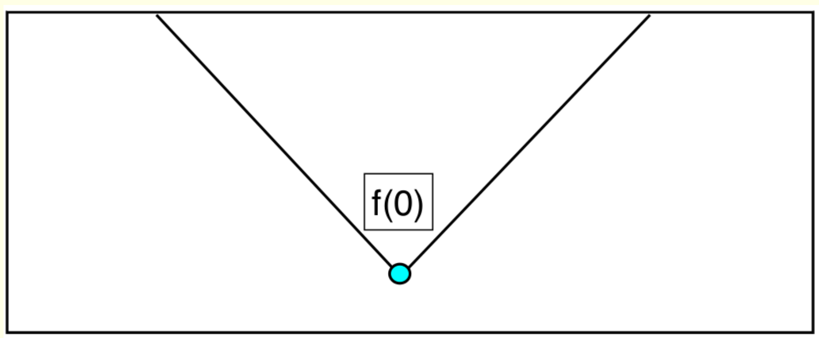

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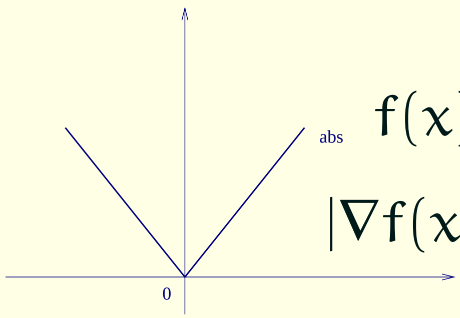

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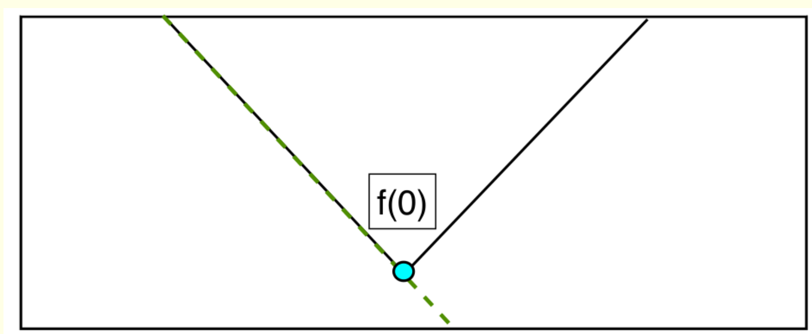

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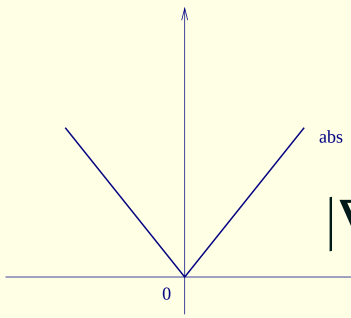

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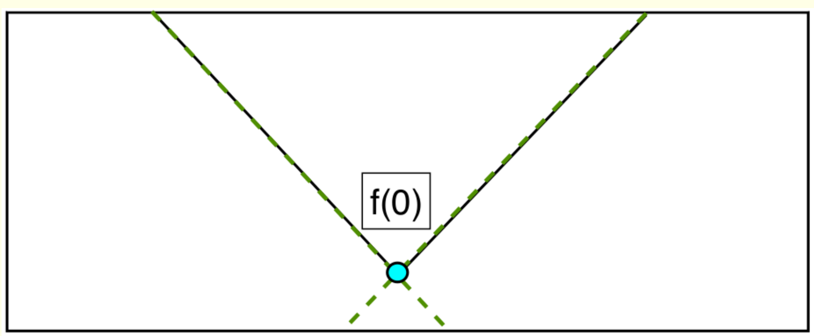

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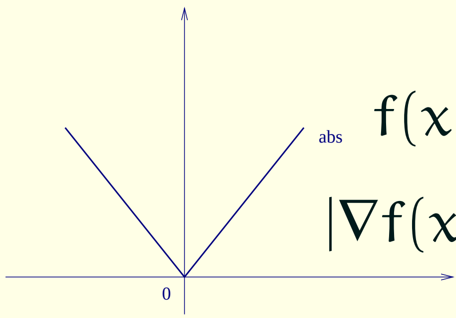

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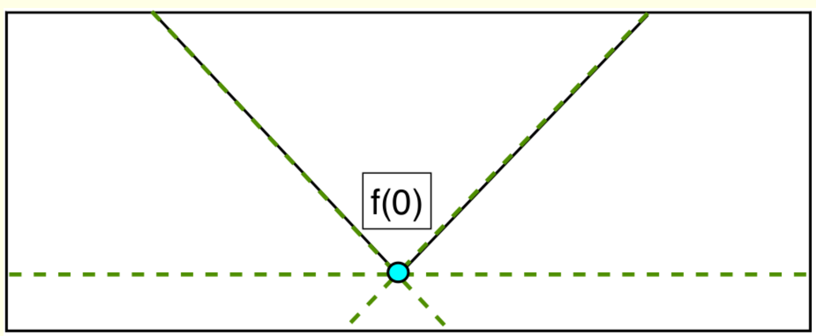

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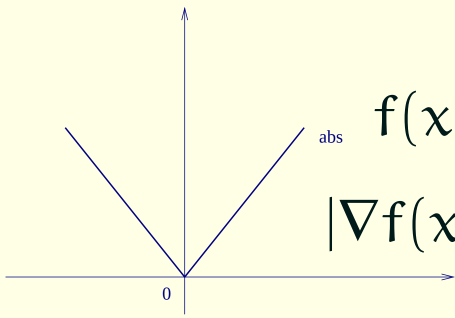

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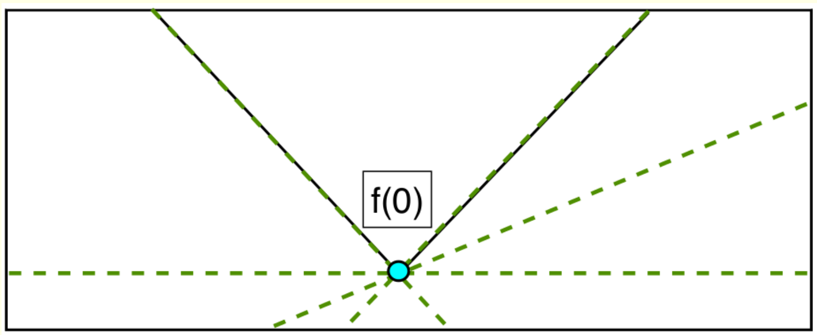

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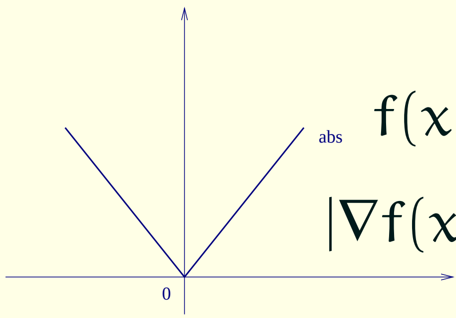

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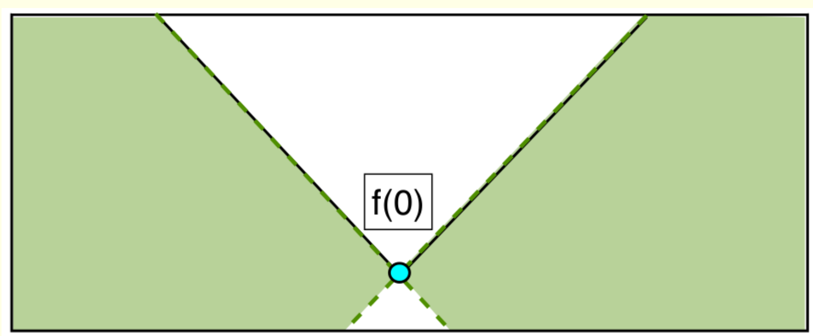

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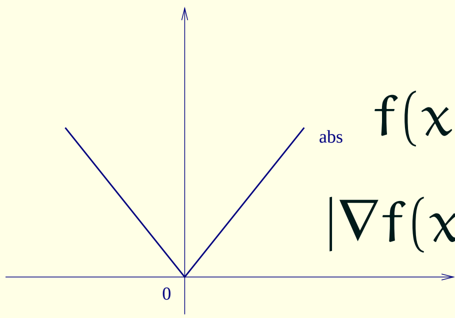

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Finite difference approximations **fail** **(no automatic differentiation)**

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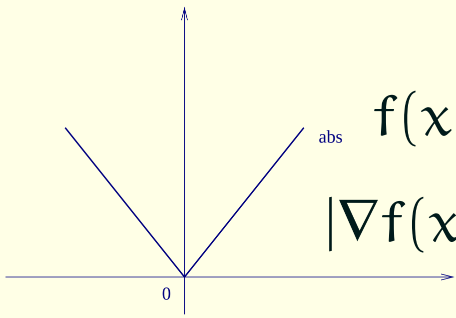

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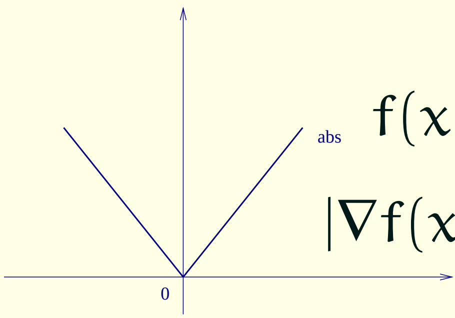
Smooth stopping test **fails**: $|\nabla f(x^k)| \leq$TOL $\qquad (\leftrightarrow |g(x^k)| \leq$TOL$)$

Finite difference approximations **fail**

Linesearches get trapped in kinks and **fail**

Why special NSO methods?

Smooth optimization methods **do not work**



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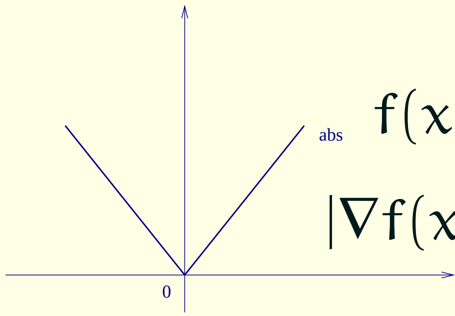

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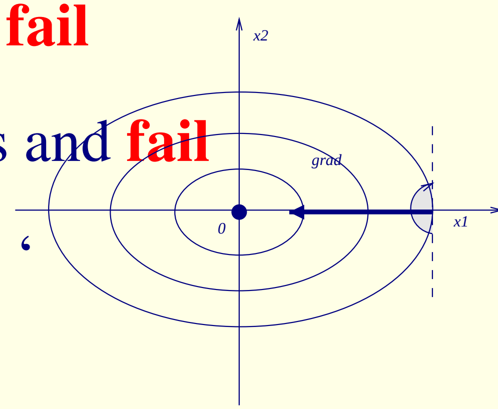
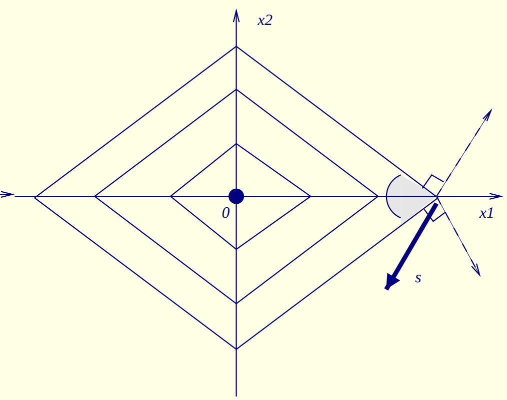
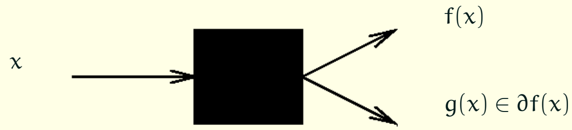




How is the oracle information used?

We look for algorithms based on information provided by an oracle



endowed with reliable stopping tests

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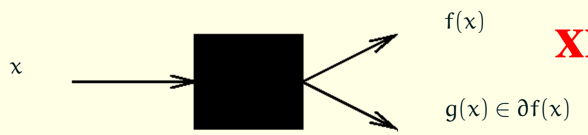

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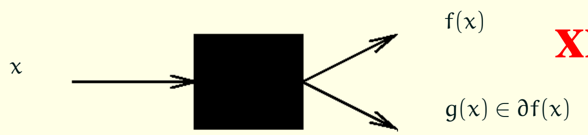

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Subgradient Methods

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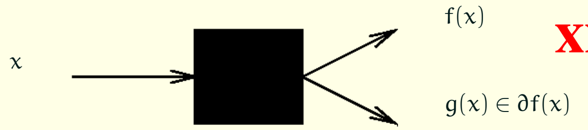

~~endowed with reliable stopping tests~~

Subgradient Methods

0 Choose x^1 and set $k = 1$.

1 Call the oracle at x^k.

2 Compute $x^{k+1} = x^k - t_k \frac{g(x^k)}{\|g(x^k)\}}$ for a suitable stepsize $t_k > 0$.

3 Make $k = k + 1$ and loop to 1.

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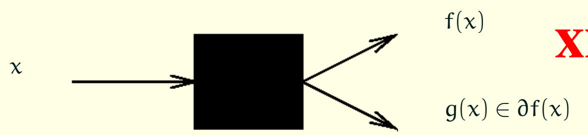

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Is this a good "recipe"?


CORAL SEA COURT
LOWER ENTRANCE
USE TOP ENTRANCE

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SG methods are

the algorithmic version

of this road sign

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…something is missing!!!

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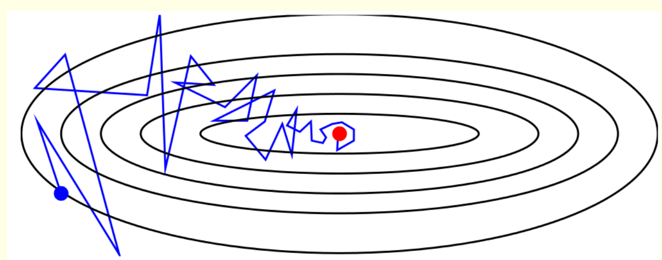

Non-monotone!

Subgradient Methods: why a "not-good" recipe

$$\begin{cases} \text{Non-monotone functional values, but converges} \\ \text{because distance to solution set decreases for } \sum t_k = +\infty, \sum t_k^2 < +\infty \\ \end{cases}$$

Subgradient Methods: why a "not-good" recipe

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Constrained case dealt with by projecting onto X: reasonable for simple X only

Subgradient Methods: why a “not-good” recipe

$$\begin{cases} \text{Non-monotone functional values, but converges} \\ \text{because distance to solution set decreases for } \sum t_k = +\infty, \sum t_k^2 < +\infty \\ \text{Lacks a stopping test} \end{cases}$$

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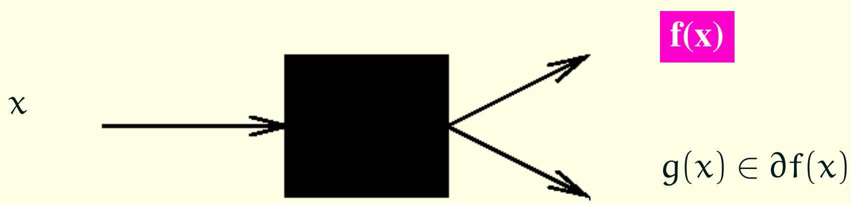

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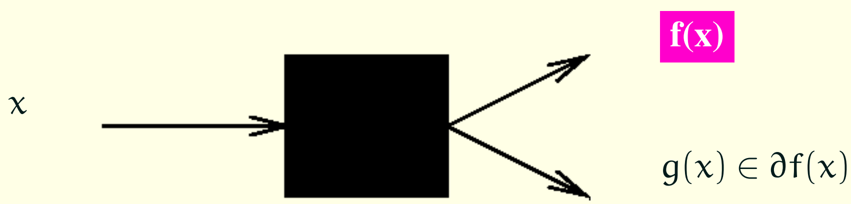

SG methods are like caipirinha without cachaça

How is the oracle information used?

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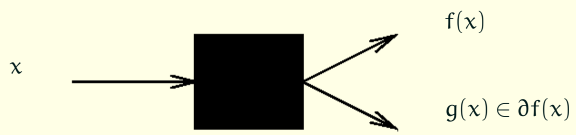

endowed with reliable stopping tests

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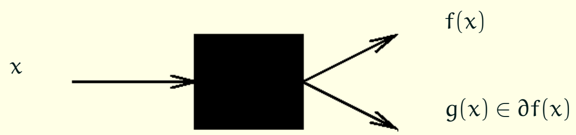

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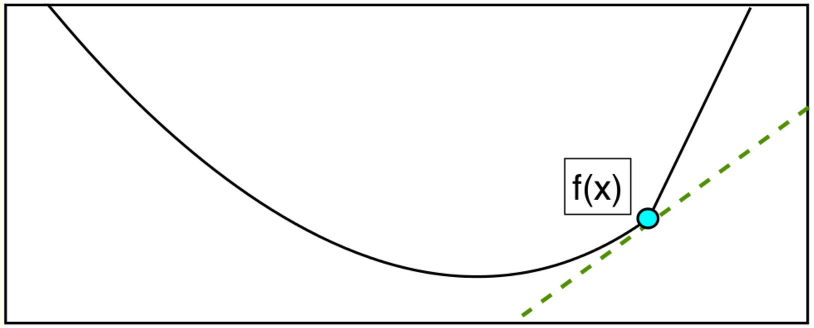
Black box information defines linearizations



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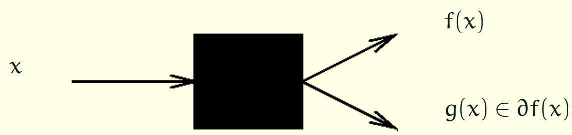

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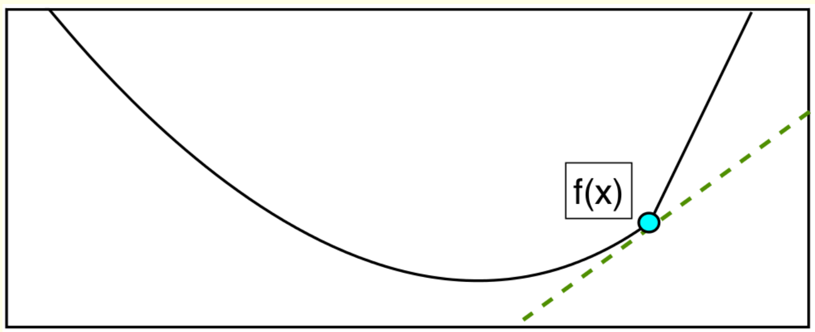
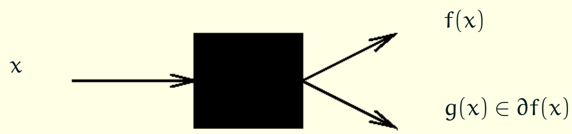


that put together create a **model M** of the function f.

The model is used to define iterates and to put in place a reliable stopping test

How is the oracle information used?

We look for algorithms based on information provided by an oracle



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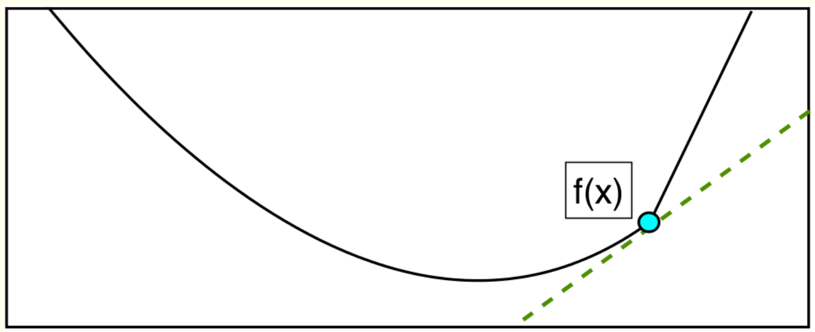
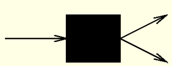
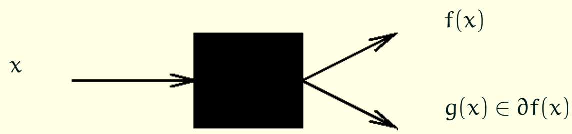


that put together create a **model M** of the function f.

x^i  $f^i = f(x^i)$, $g^i = g(x^i)$ $\Longrightarrow$ $f^i + {g^i}^\top(x - x^i)$

How is the oracle information used?

We look for algorithms based on information provided by an oracle



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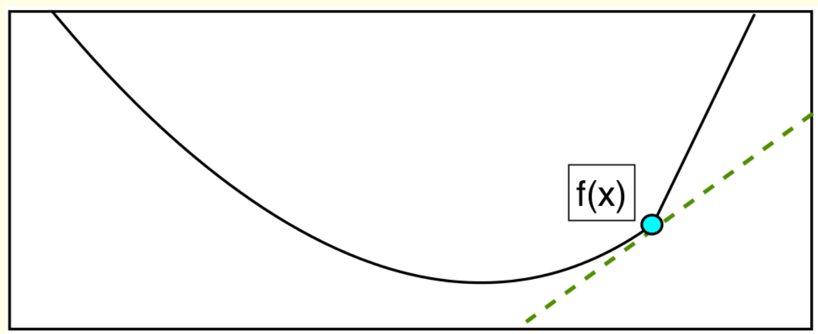

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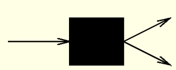

$$f^i = f(x^i)$$
$$g^i = g(x^i)$$

$$\Longrightarrow \mathbf{M}(x) = \max_i \{ f^i + {g^i}^\top (x - x^i) \}$$

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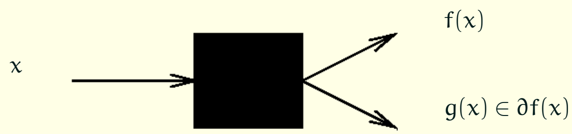

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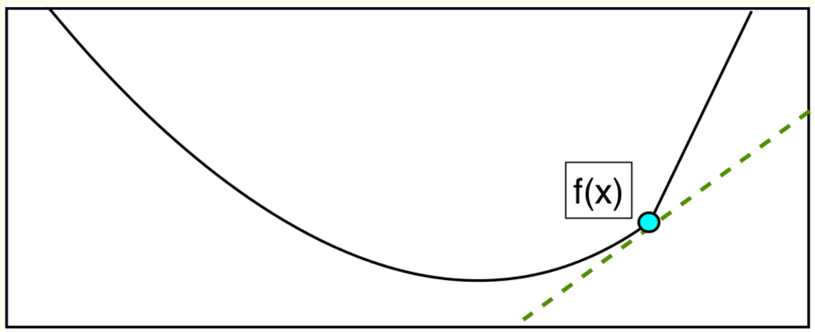

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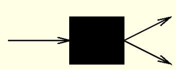

$$f^i = f(x^i)$$

$$g^i = g(x^i)$$

$$\Longrightarrow \mathbf{M}(x) = \max_i \{ f^i + g^{i\top}(x - x^i) \}$$

(just an example, many other models are possible)

Cutting-plane methods

To minimize f (unavailable in an explicit manner), minimize its model $\mathbf{M}(x) = \max_i \left\{ f^i + g^{i\top}(x - x^i) \right\}$

Improve the model at each iteration

Cutting-plane methods

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Improve the model at each iteration:

$$\begin{aligned} \mathbf{M}_{k+1}(x) &= \max_{i \le k+1} \left\{ f^i + g^{i\top}(x - x^i) \right\} \\ &= \max\Big(\mathbf{M}_k(x), f^{k+1} + g^{k+1\,\top}(x - x^{k+1}) \Big) \\ &\quad \textbf{where } x^{k+1} \textbf{ minimizes } \mathbf{M}_k \end{aligned}$$

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Instead of $x^* \in \arg\min f(x)$ at one shot

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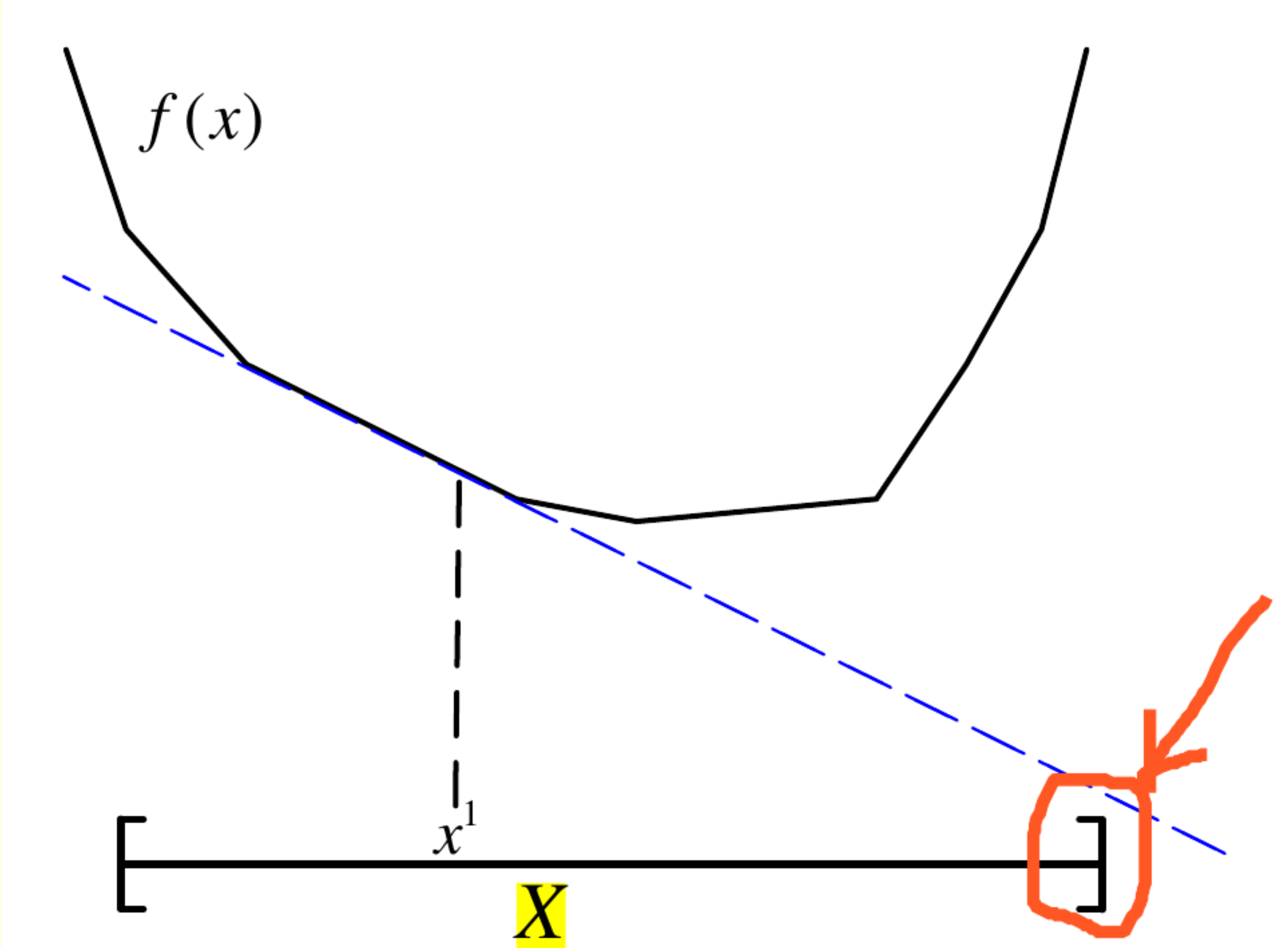
Instead of $x^* \in \arg\min f(x)$ at one shot,

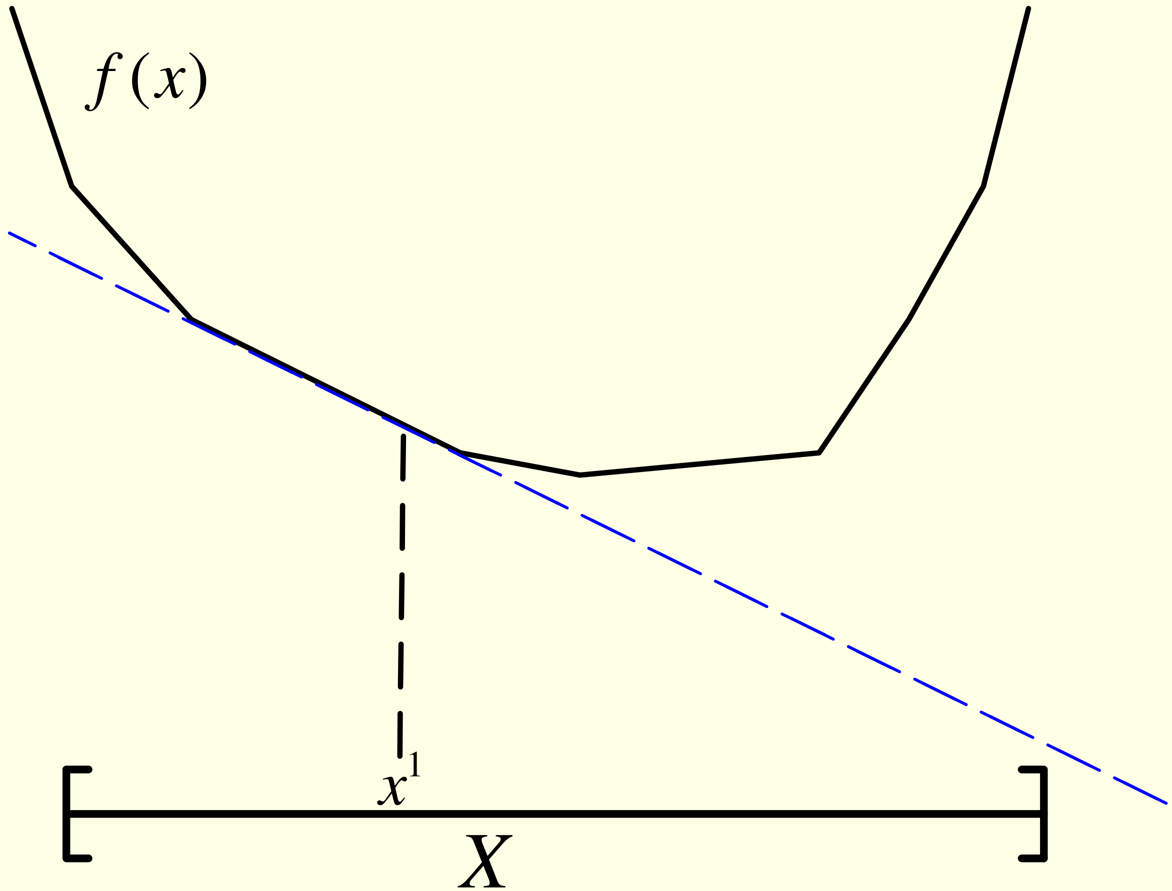
$x^{k+1} \in \arg\min \mathbf{M}_k(x)$ **iteratively**

Cutting-plane methods



Artificial bounding at least for the first iterations

Cutting-plane methods



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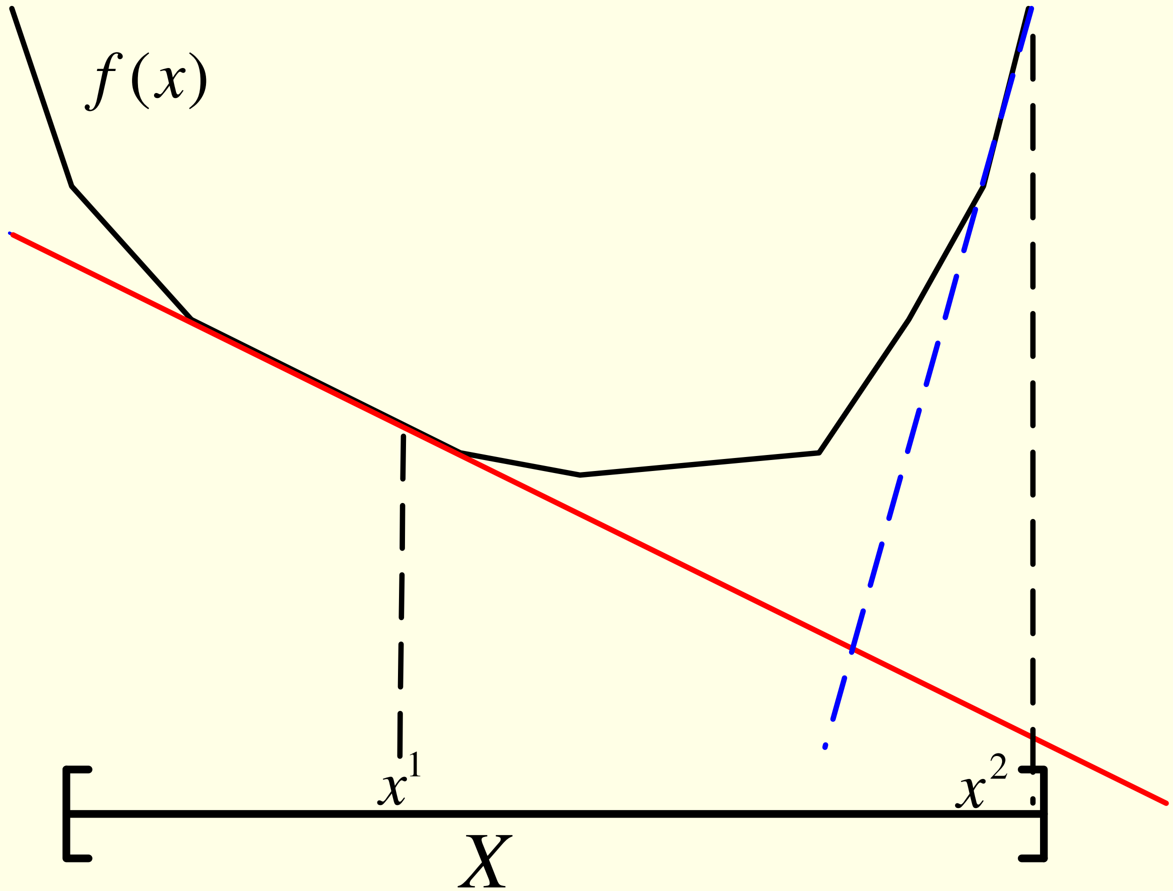

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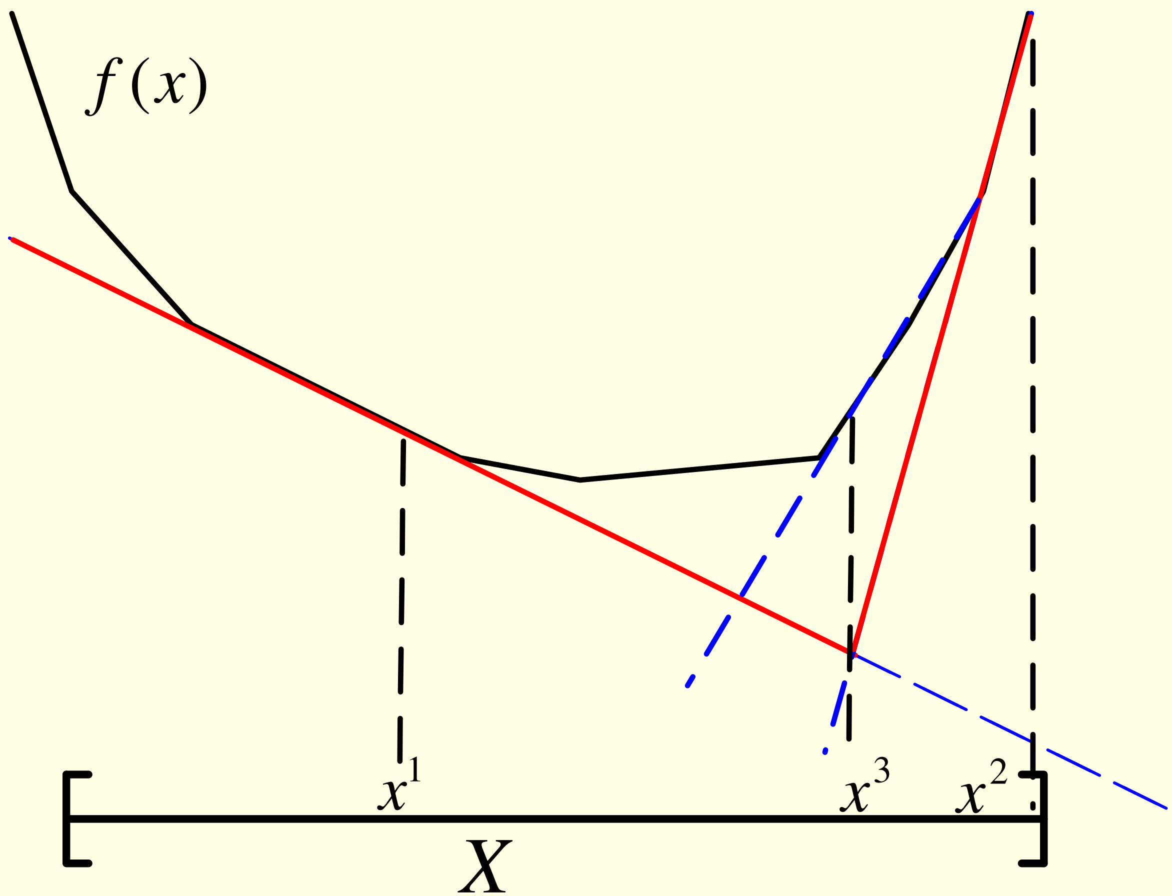

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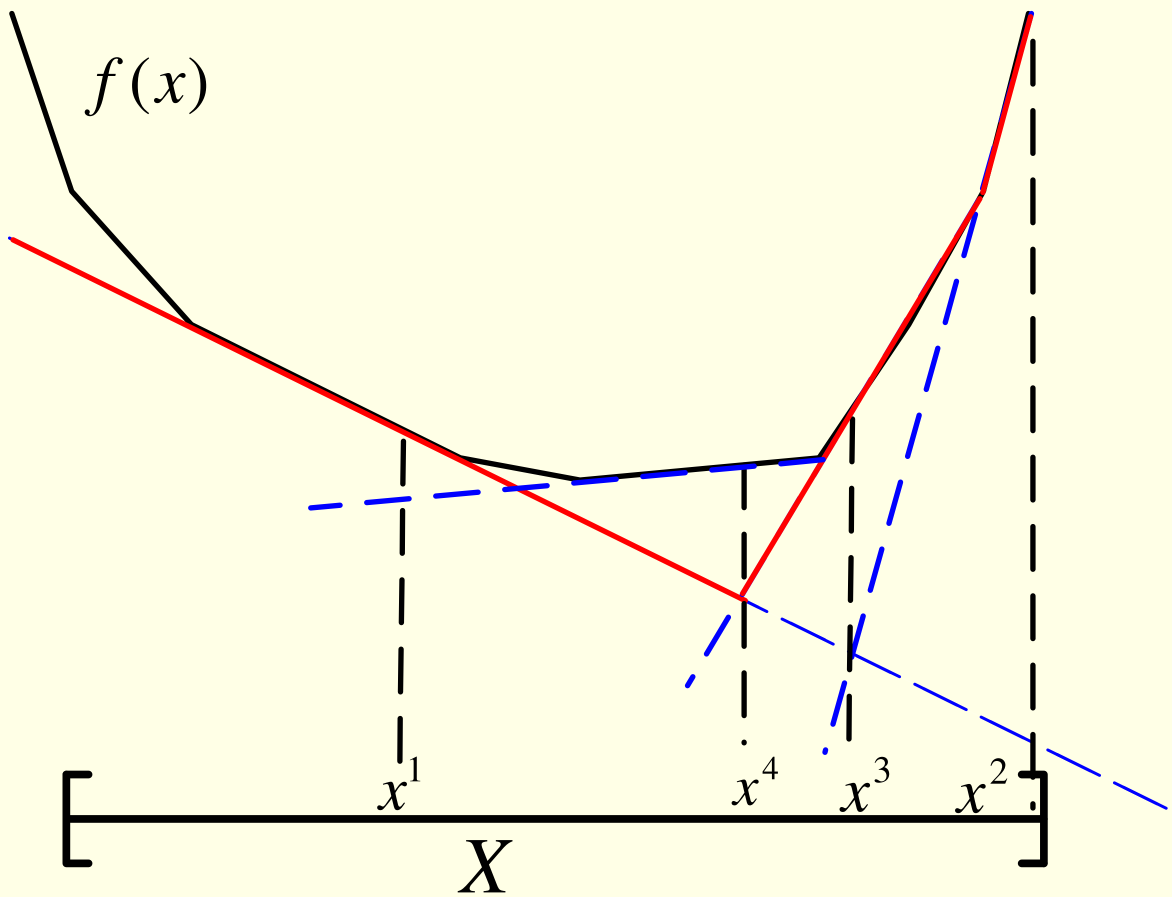

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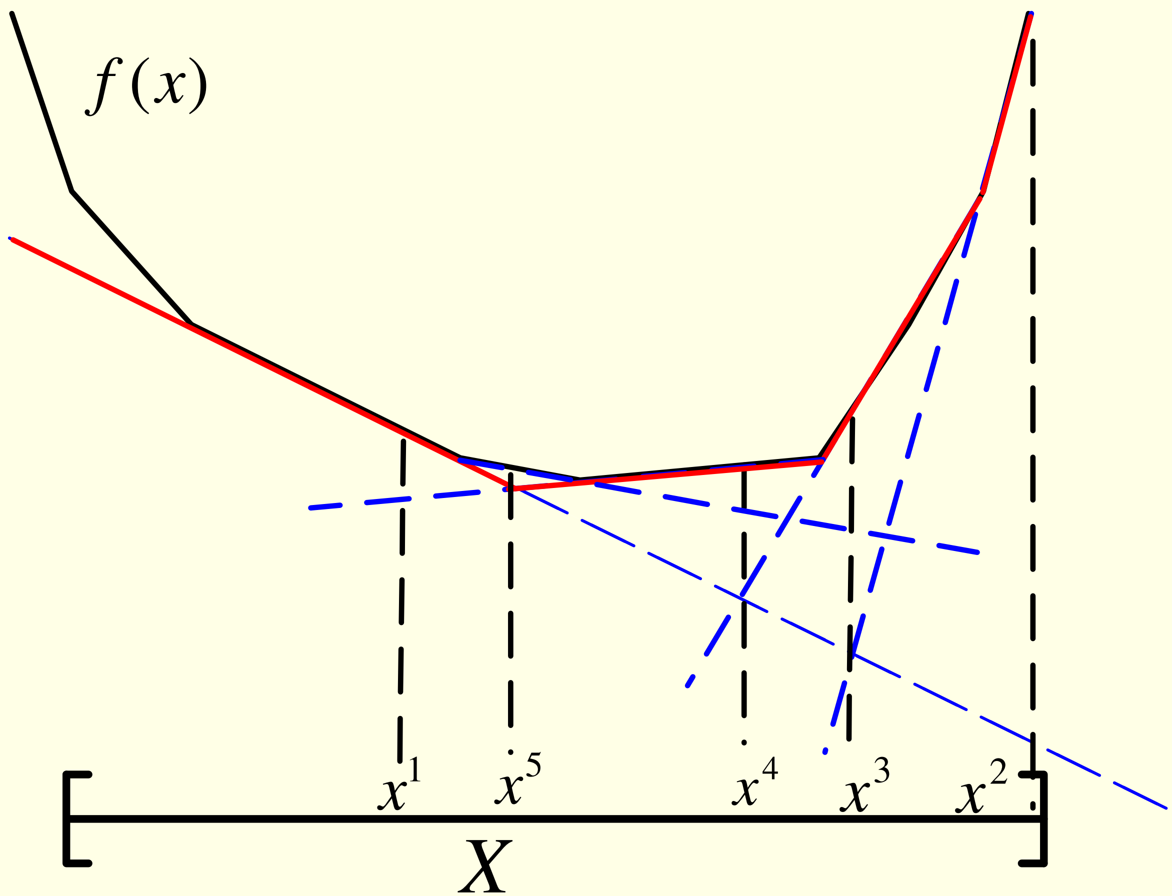

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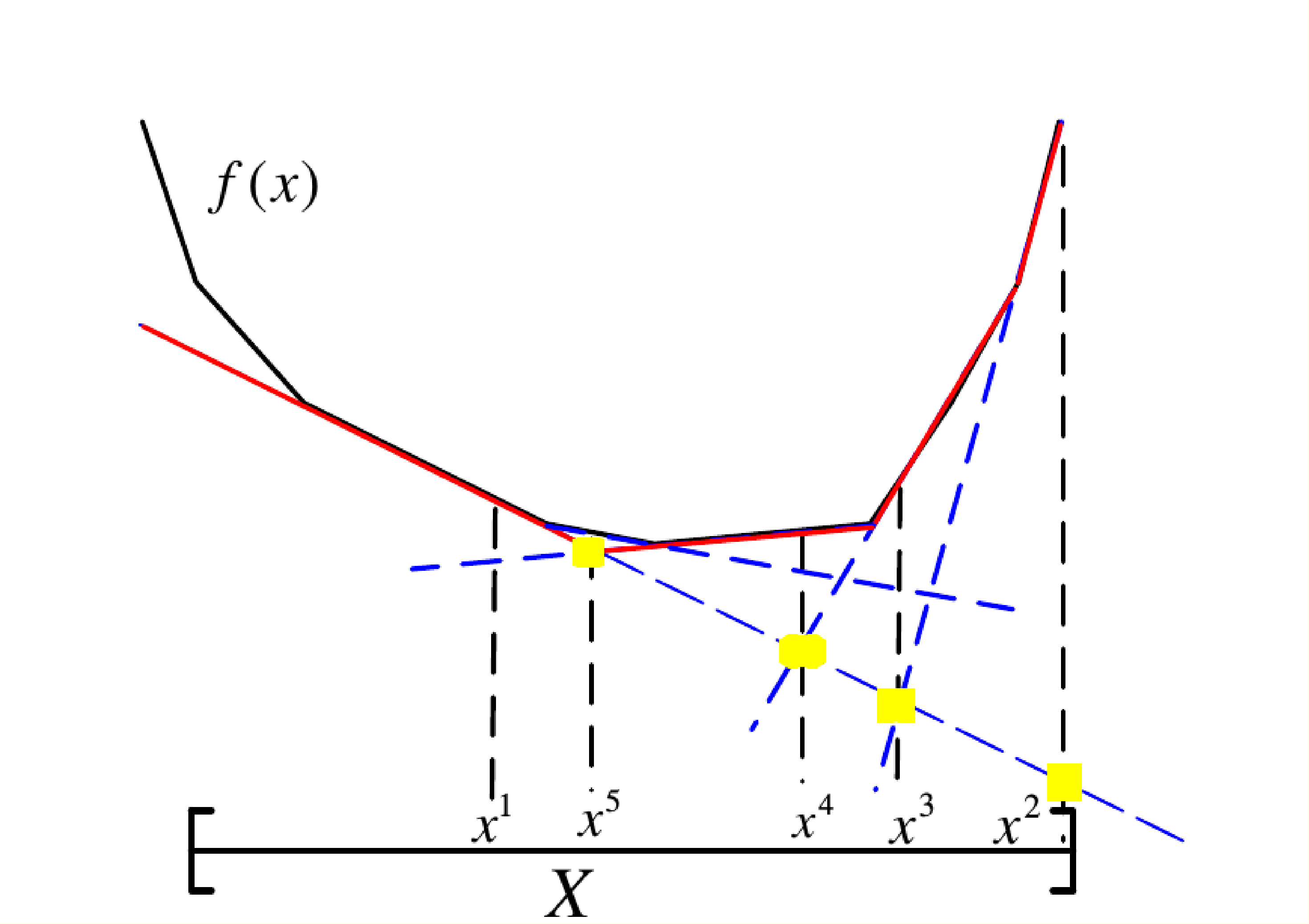
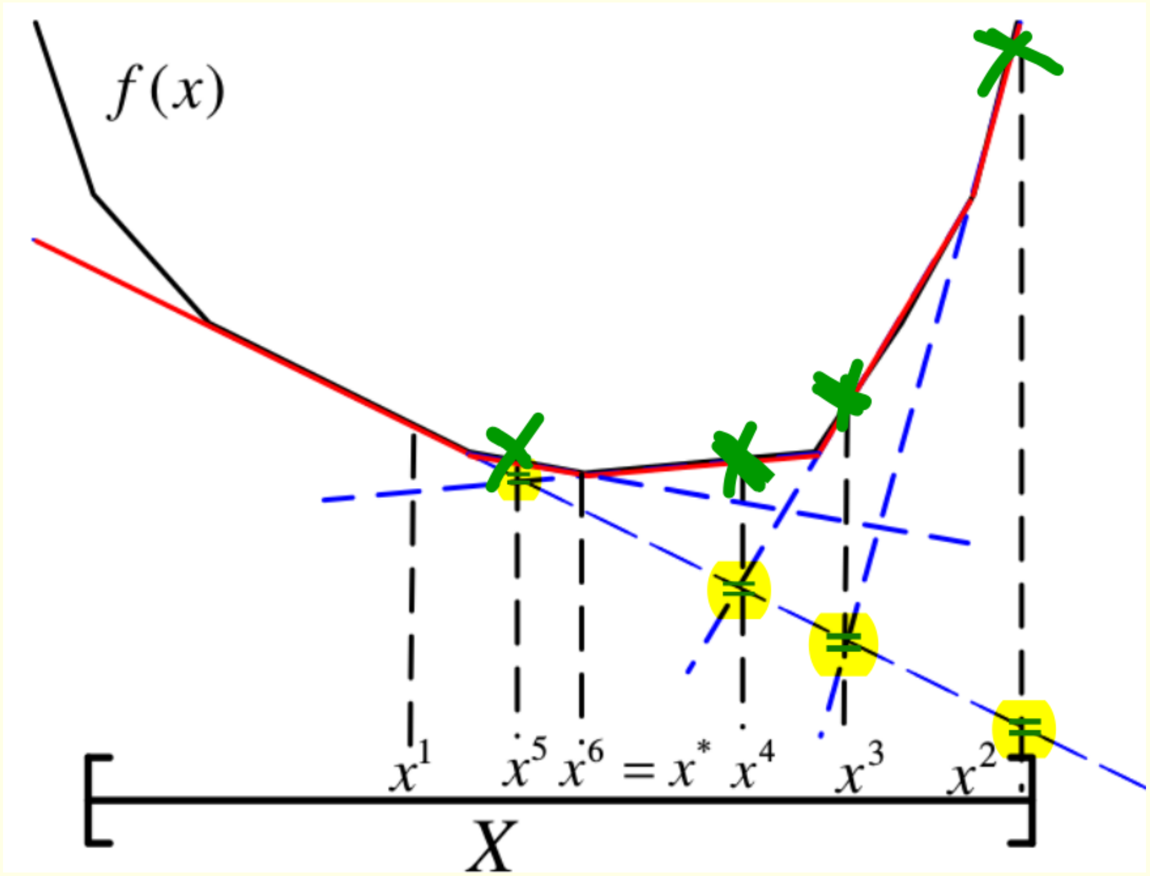


$\{\mathbf{M}_k(x^{k+1})\}$ increases

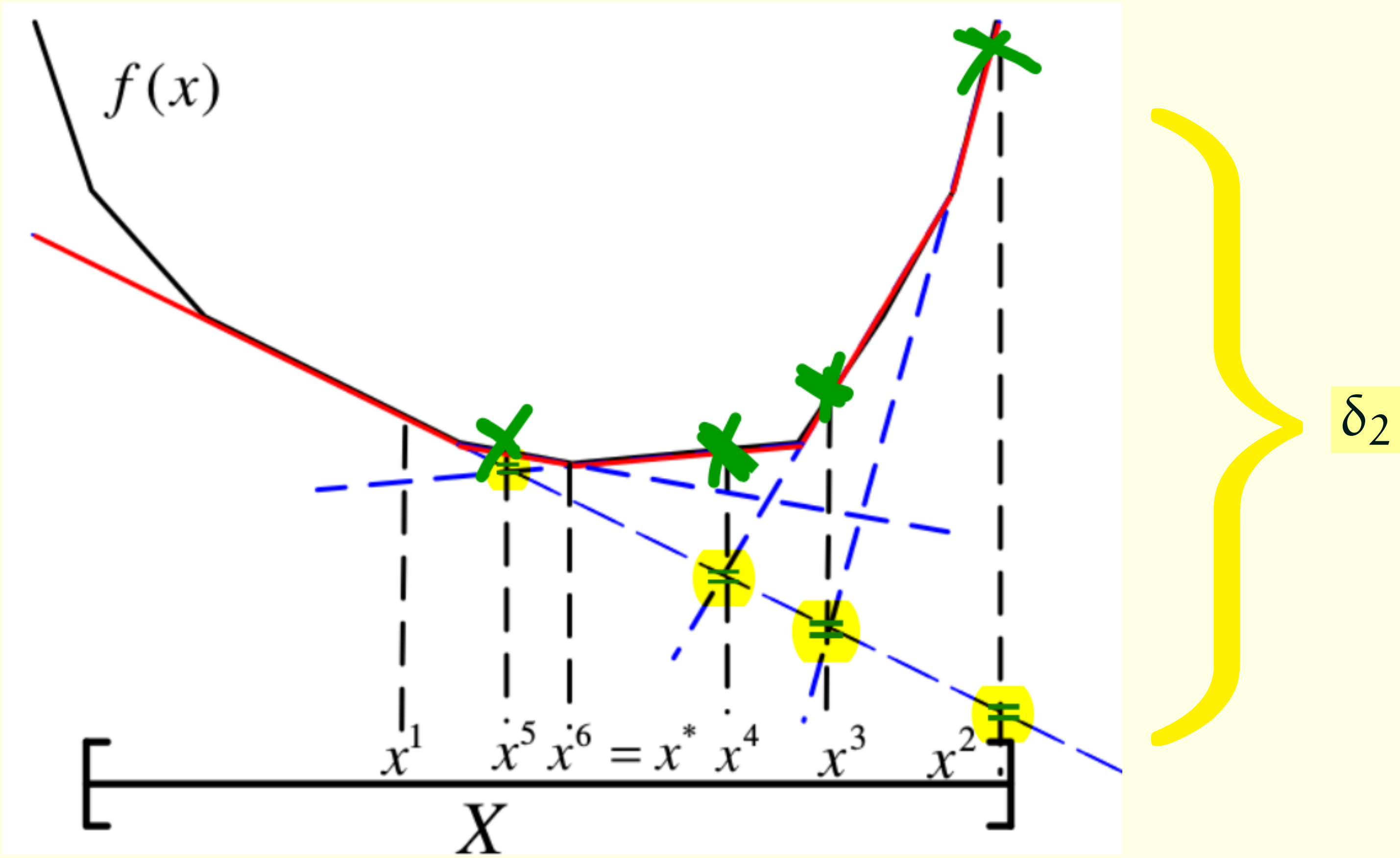
Cutting-plane methods



$\{\mathbf{M}_k(x^{k+1})\}$ increases but not necessarily the functional values:

$f(x^5) > f(x^4)$

Cutting-plane methods



$\{\mathbf{M}_k(x^{k+1})\}$ increases but not necessarily the functional values: $f(x^5) > f(x^4)$. Stopping test measures $\delta_k := f(x^k) - \mathbf{M}_{k-1}(x^k)$

Cutting-plane Methods

0 Choose x^1 and set $k = 1$.

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Cutting-plane Methods

0 Choose x^1 and set $k = 1$.

1 Call the oracle at x^k. **If** $f(x^k) - \mathbf{M}_{k-1}(x^k) \le tol$ **STOP**

2 Compute $x^{k+1} \in \arg\min_X \mathbf{M}_k(x)$

3 $\mathbf{M}_{k+1}(\cdot) = \max\left(\mathbf{M}_k(\cdot), f^k + g^{k\top}(\cdot - x^k)\right)$, $k = k+1$, loop to 1.

CP methods are

an improved algorithmic version

of the Aussie sign

a better recipe