Risk-constrained dynamic asset allocation via stochastic dual dynamic programming

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Objective and contributions

Develop a realistic and computationally tractable stochastic dynamic asset allocation model considering time consistent and intuitive risk constraints, time dependence and transaction cost.

Main Contributions:

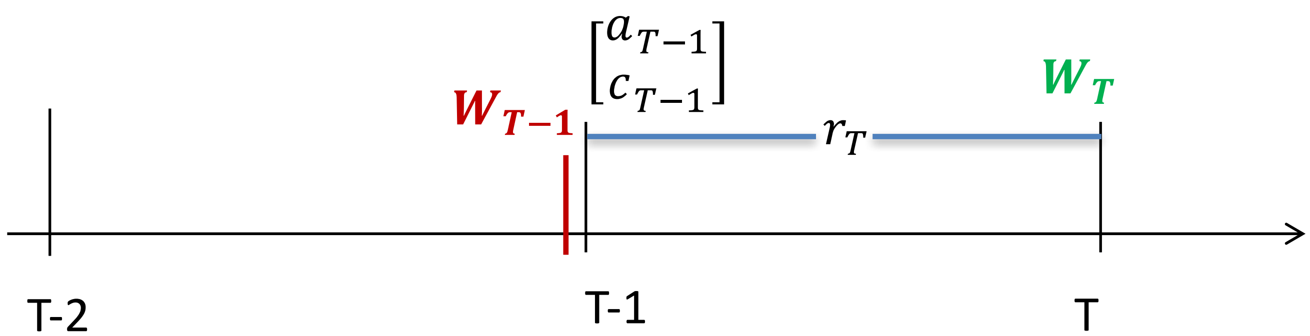
- Time consistent risk-constrained stochastic dynamic asset allocation model
  - Realistic: transaction costs and time dependence
  - Risk aversion: intuitive user defined loss limit
  - Computationally tractable: SDDP with Markovian policy

No transactional cost and time independence

$$Q_t(W_t) = \max_{a_t, c_t, d_t \geq 0} \phi\left[Q_{t+1}\left(\overbrace{c_t + \sum\nolimits_{i \in \mathcal{A}} (1 + r_{i,t+1}) a_{i,t}}^{\boldsymbol{W_{t+1}}}\right)\right]$$

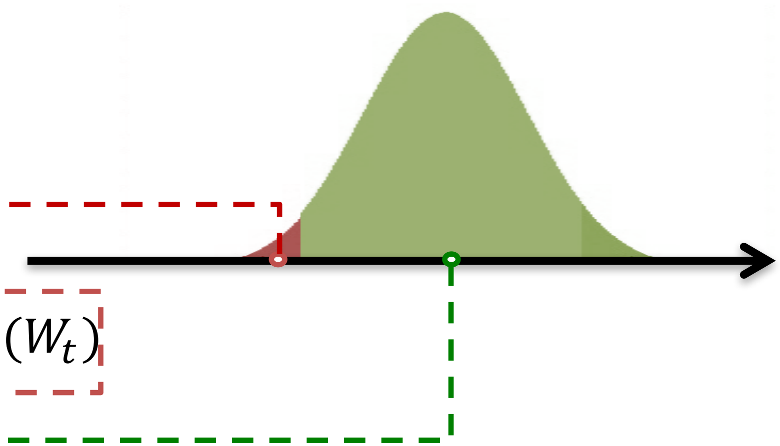
$$s.t. \qquad c_t + \sum\nolimits_{i \in \mathcal{A}} a_{i,t} = W_t$$

Where, $Q_T(\boldsymbol{W_T}) = \boldsymbol{W_T}$



Time consistent (recursive) model

- Usually the risk measure is the convex combination of the expect return and the CVaR



$$\phi(W_t) = (1 - \lambda)\mathbb{E}[W_t] + \lambda CVaR_\alpha(W_t)$$

- Economicinterpretation: certainequivalent
  – Rudloff, Street, Valladão (2014)
- Problem: Howshouldwe define λ?

Risk constrained model

$$V_t(W_t) = \max_{a_t, c_t \geq 0} \quad \mathbb{E}\left[V_{t+1}\left(c_t + \sum_{i \in \mathcal{A}} (1 + r_{i,t+1}) a_{i,t}\right)\right]$$

$$s.t. \quad \rho_t\left[\sum_{i \in \mathcal{A}} r_{i,t+1} a_{i,t}\right] \leq \gamma W_t$$

$$c_t + \sum_{i \in \mathcal{A}} a_{i,t} = W_t$$

- Intuitive risk averse parameter γ
- Relative complete recourse for $\gamma \geq 0$.
  - One can always allocate in cash
- Positively homogeneous

If $W_t \geq 0$, $V_t(W_t) = W_t \cdot V_t(1)$

Myopic Solution

$$V_t(W_t) = \max_{a_t, c_t \geq 0} \mathbb{E}\left[V_{t+1}\left(c_t + \sum_{i \in \mathcal{A}} (1 + r_{i,t+1}) a_{i,t}\right)\right]$$

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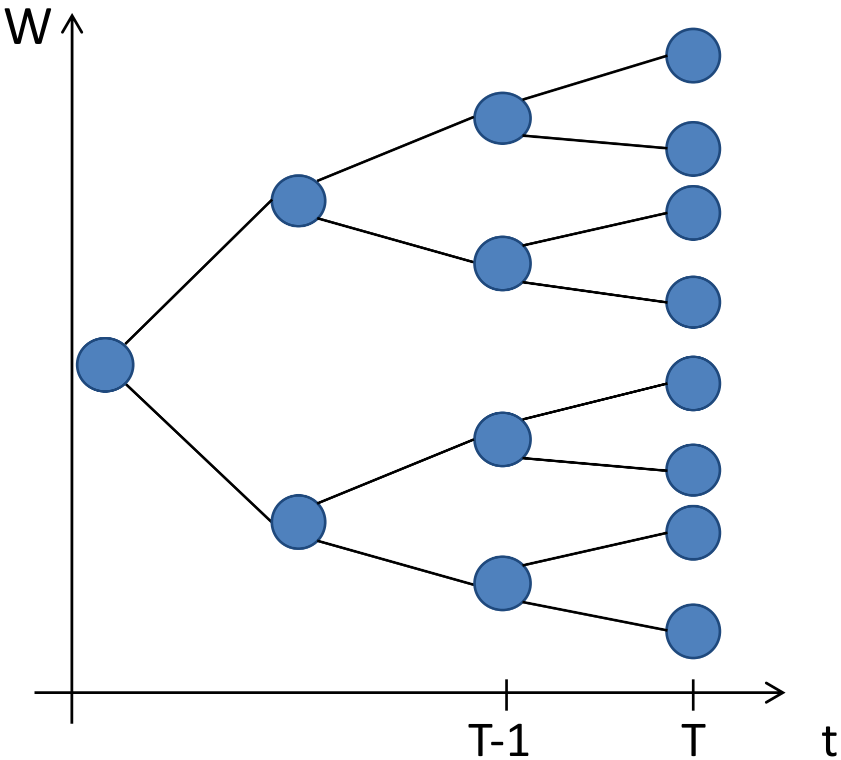

Myopic Solution

$$V_t(W_t) = \max_{a_t, c_t \geq 0} \mathbb{E}\left[c_t + \sum_{i \in \mathcal{A}} (1 + r_{i,t+1}) a_{i,t}\right]$$

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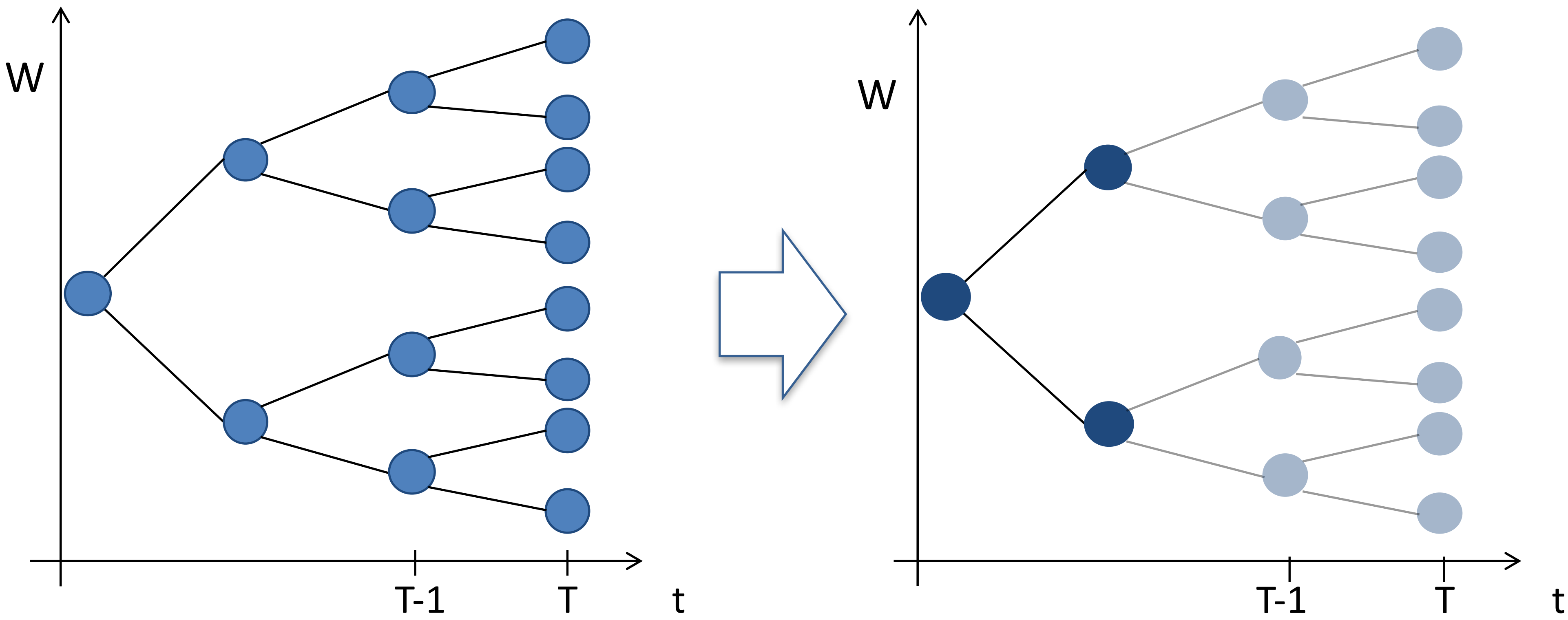


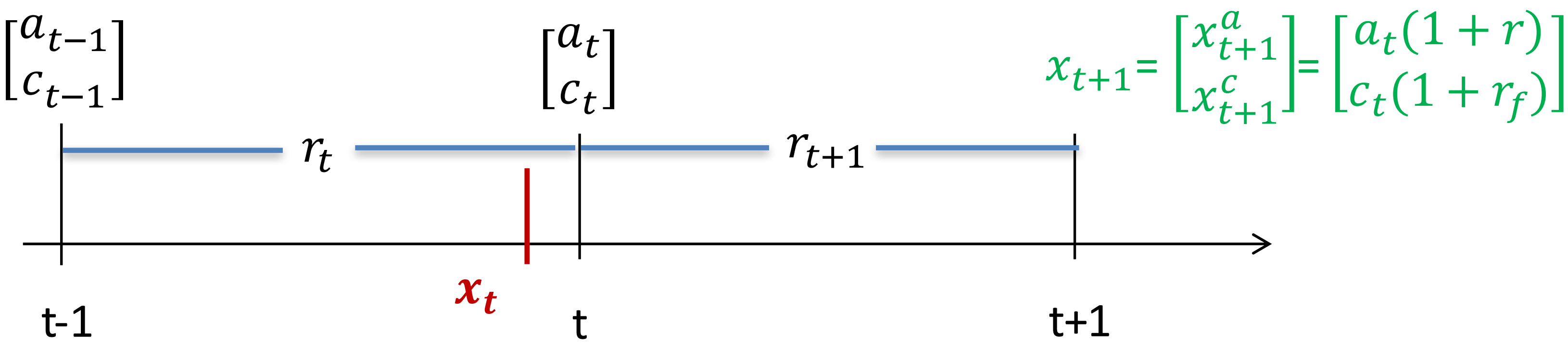
Transactional cost and time independence

$$V_t(x_t) = \max_{a_t, c_t, d_t \geq 0} \quad \mathbb{E}[V_{t+1}(x_{t+1})]$$

$$s.t. \quad \rho_t\left[\sum_i \left(r_{i,t+1} a_{i,t} - \tau\left(d_{i,t}^+ + d_{i,t}^-\right)\right)\right] \leq \gamma\left(x_t^c + \sum_i x_{i,t}^a\right)$$

$$a_{i,t} = x_{i,t}^a + d_{i,t}^+ - d_{i,t}^-, \forall i \in \mathcal{A}$$

$$c_t = x_t^c - (1+\tau)\sum_i d_{i,t}^+ + (1-\tau)\sum_i d_{i,t}^-$$



Simplifying notation

$$V_T(x_T) = x_t^c + \sum_i x_{i,t}^a$$

$$V_t(x_t) = \max_{u \in U(x_t)} \mathbb{E}\left[V_{t+1}\left(x_{t+1}(u)\right)\right], \qquad \forall t \in \{0, \dots, \mathrm{T} - 1\}$$

$$U(x) = U\left((x^c, x^a)\right)$$

$$= \left\{ (\mathrm{c}, \mathrm{a}, d^+, d^-) \in \mathbb{R}_+^{3N+1} \;\middle|\; \begin{array}{c} \rho_t \left[\sum_i \left(r_{i,t+1} a_{i,t} - \tau\left(d_{i,t}^+ + d_{i,t}^-\right)\right)\right] \leq \gamma\left(x_t^c + \sum_i x_{i,t}^a\right) \\ c = x^c - (1+\tau) \sum_{i \in \mathcal{A}} d_i^+ + (1-\tau) \sum_{i \in \mathcal{A}} d_i^- \\ a_i = x_i^a + d_i^+ - d_i^-, \; \forall i \in \mathcal{A} \end{array} \right\}$$

$$u = (c, a_1, \dots a_N, d_1^+, \dots, d_N^+, d_1^-, \dots, d_N^-)'$$

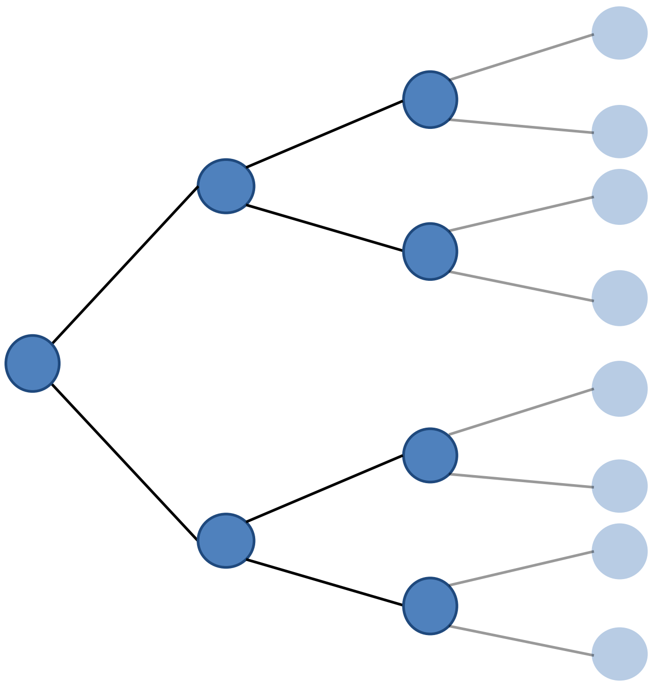
$$W_T(u) = W_T(c, a) = c + \sum_{i \in \mathcal{A}} (1 + r_{i,T}) a_i$$

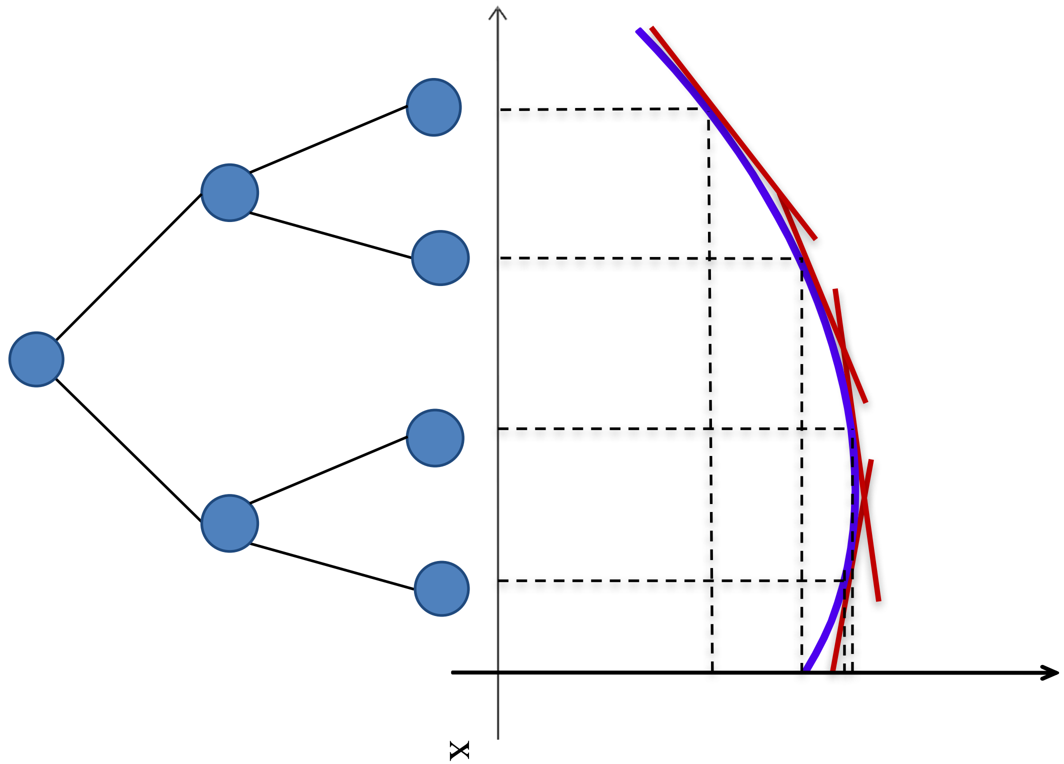
$$x_{t+1}(u) = x_{t+1}(c, a) = \left(c_t, (1 + r_{1,t+1}) a_{1,t}, \dots, (1 + r_{N,t+1}) a_{N,t}\right)'$$

Transactional cost and time independence

- Solution algorithm: SDDP

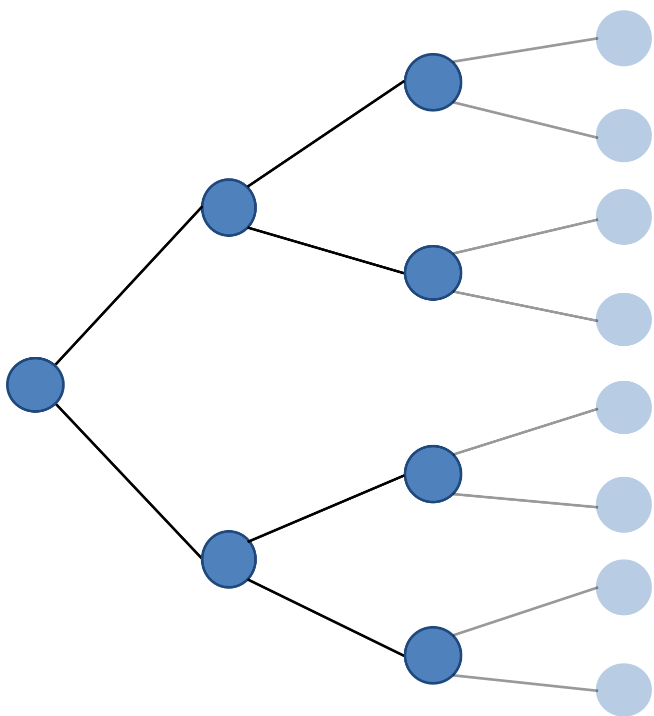
$$V_t(x_t) = \max_{u \in U(x_t)} \mathbb{E}\big[V_{t+1}\big(x_{t+1}(u)\big)\big], \qquad \forall t \in \{0, \dots, \mathrm{T}-1\}$$

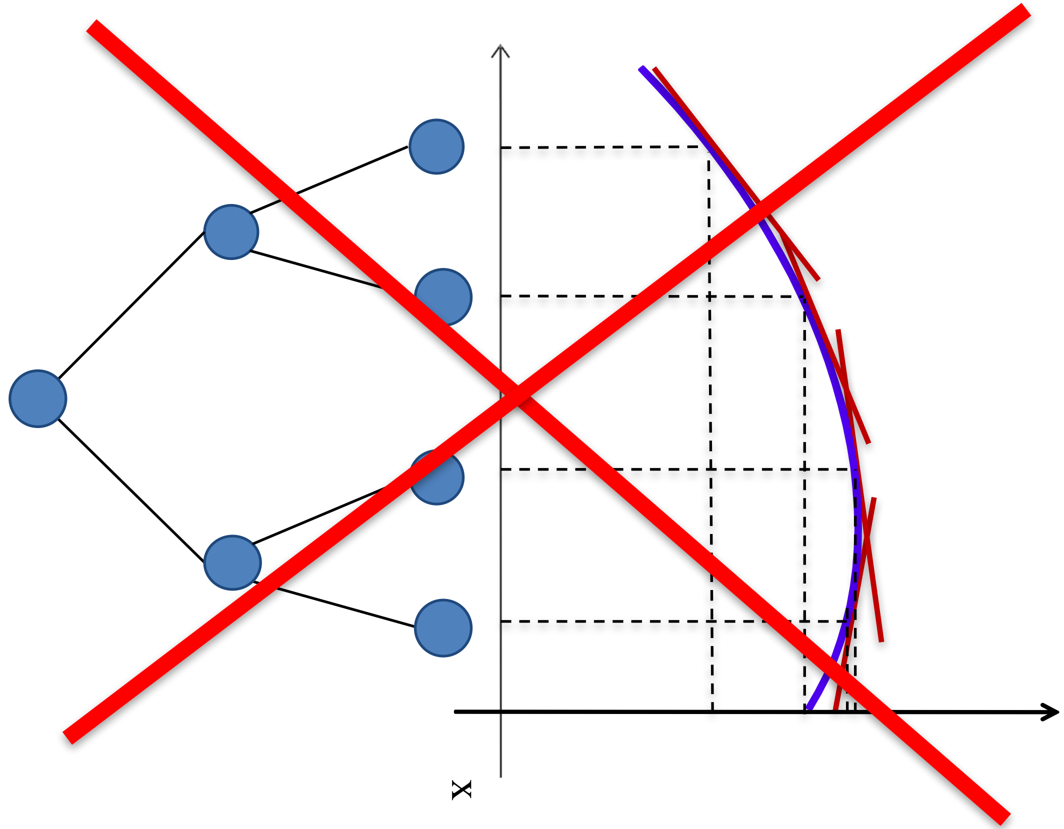




Transactional cost and time ***dependence***

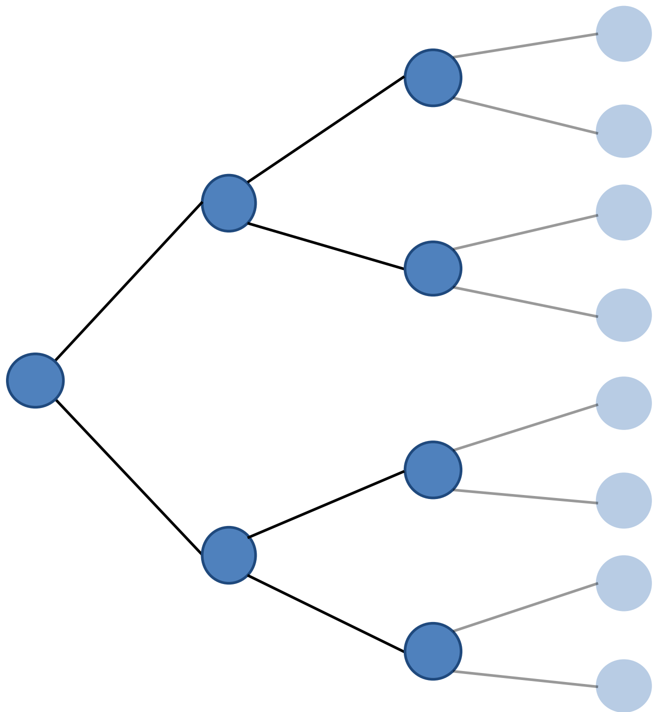
$$V_t(x_t, r_{[t]}) = \max_{u \in U(x_t)} \mathbb{E}\left[V_{t+1}(x_{t+1}(u)) \middle| r_{[t]}\right], \qquad \forall t \in \{0, \ldots, T-1\}$$

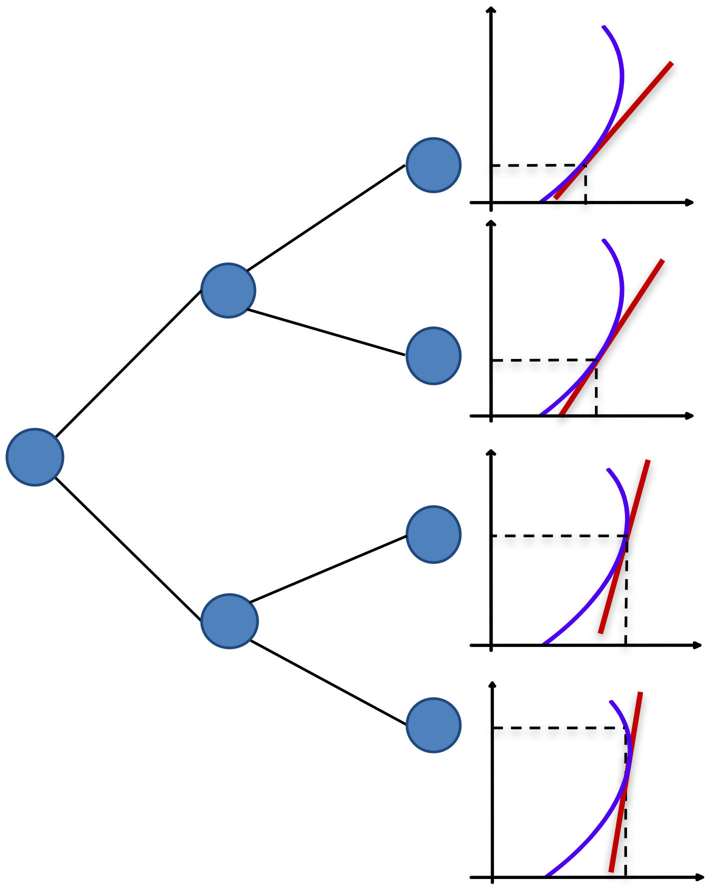




Transactional cost and time ***dependence***

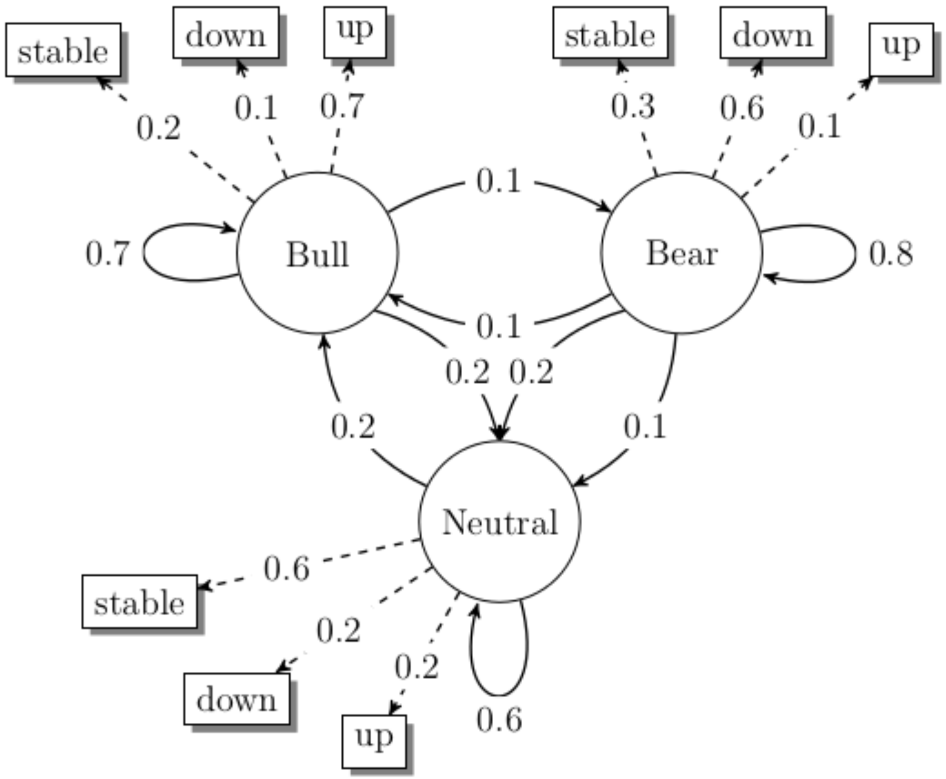
$$V_t\left(x_t, r_{[t]}\right) = \max_{u \in U(x_t)} \mathbb{E}\left[V_{t+1}\left(x_{t+1}(u)\right) \middle| r_{[t]}\right], \qquad \forall t \in \{0, \dots, \mathrm{T}-1\}$$





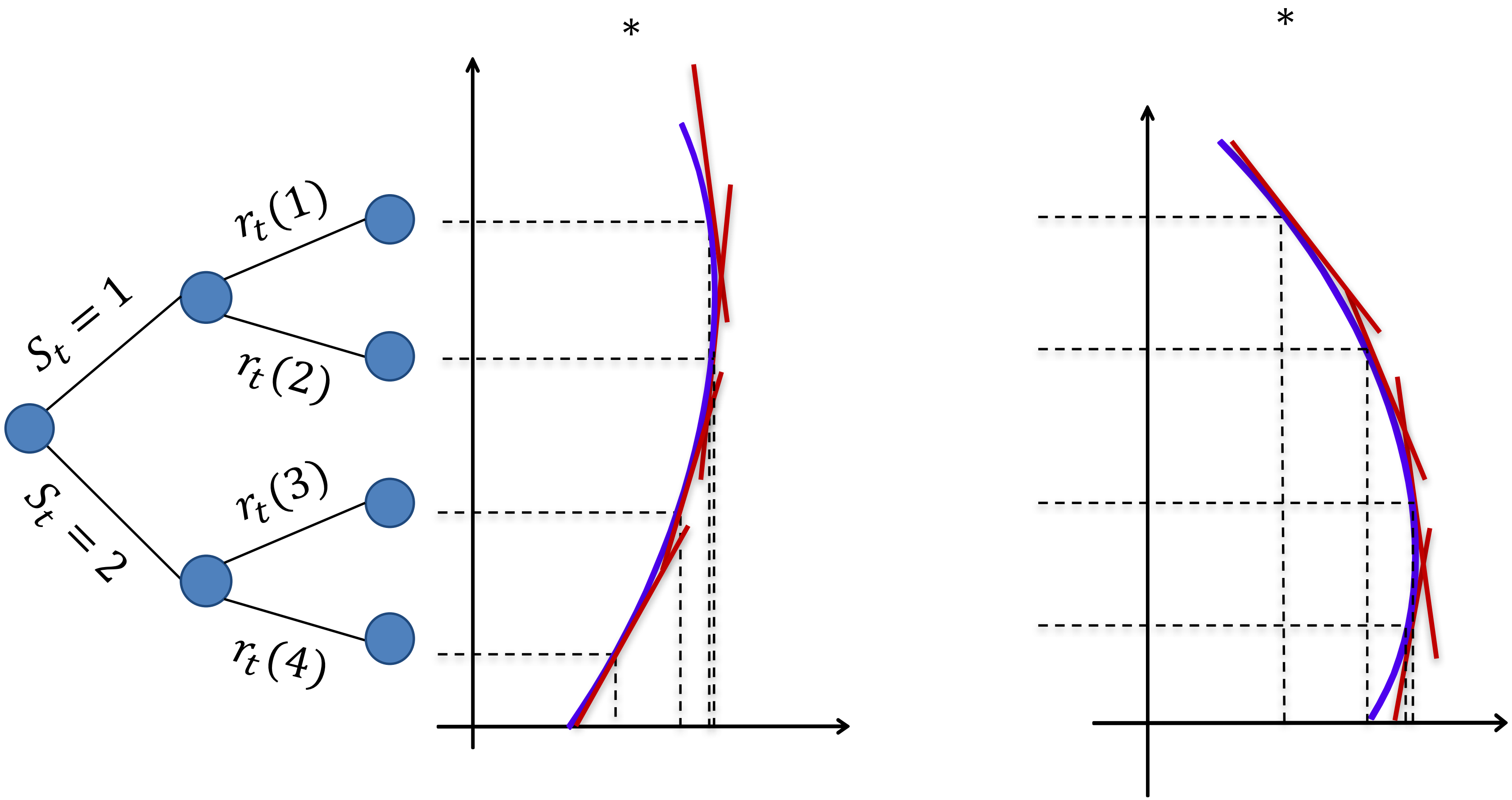
Transactional cost and Markov ***dependence***

$$V_t^k(x_t) = \max_{u \in U(x_t)} \sum_{j=1}^{K} \mathbb{E}\left[V_{t+1}^k(x_{t+1}(u)) \middle| S_{t+1} = j\right] \mathbb{P}(S_{t+1} = j | S_t = k)$$

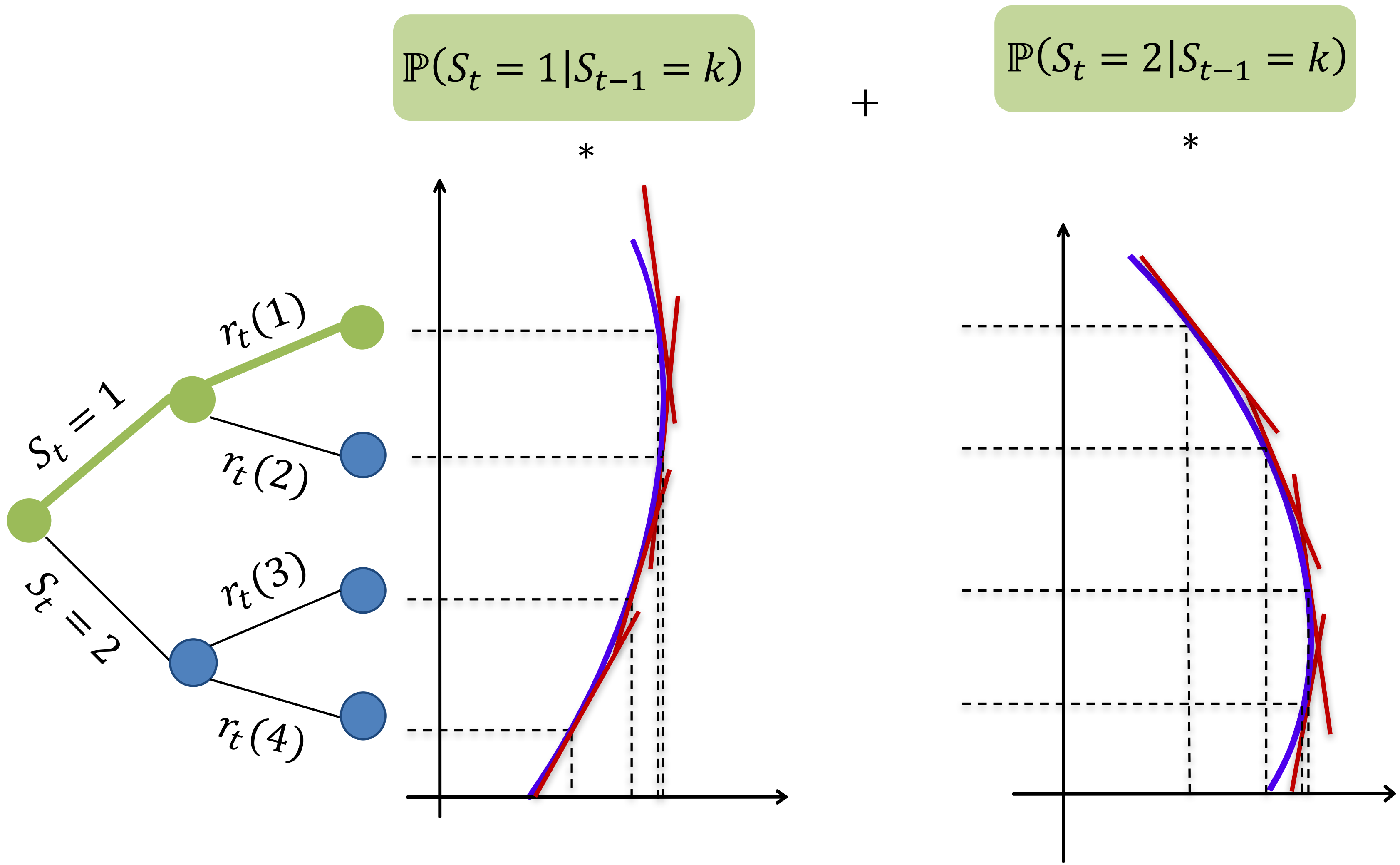


Transactional cost and Markov ***dependence***

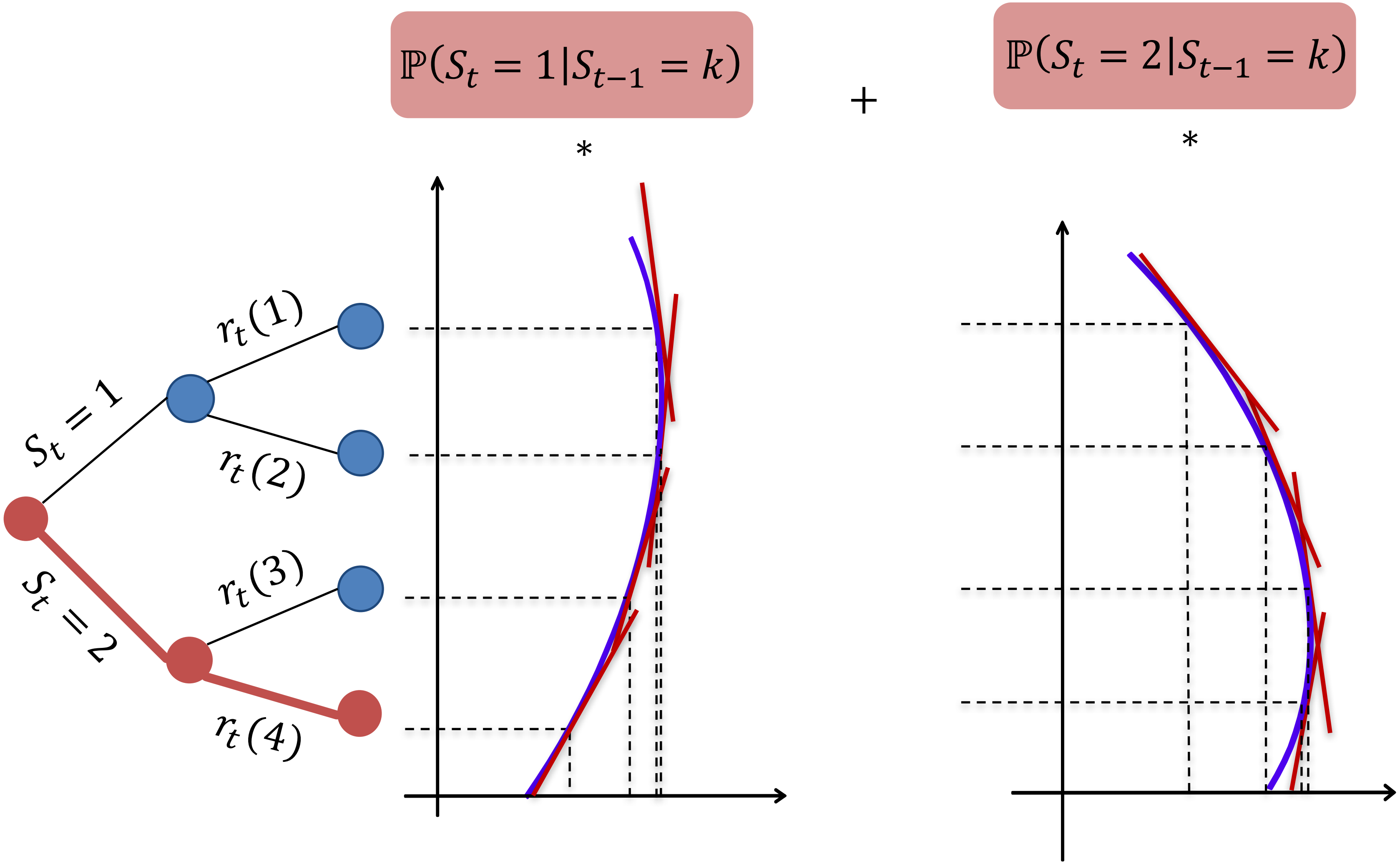
$\mathbb{P}(S_t = 1 | S_{t-1} = k)$ + $\mathbb{P}(S_t = 2 | S_{t-1} = k)$



Transactional cost and Markov ***dependence***



Transactional cost and Markov ***dependence***

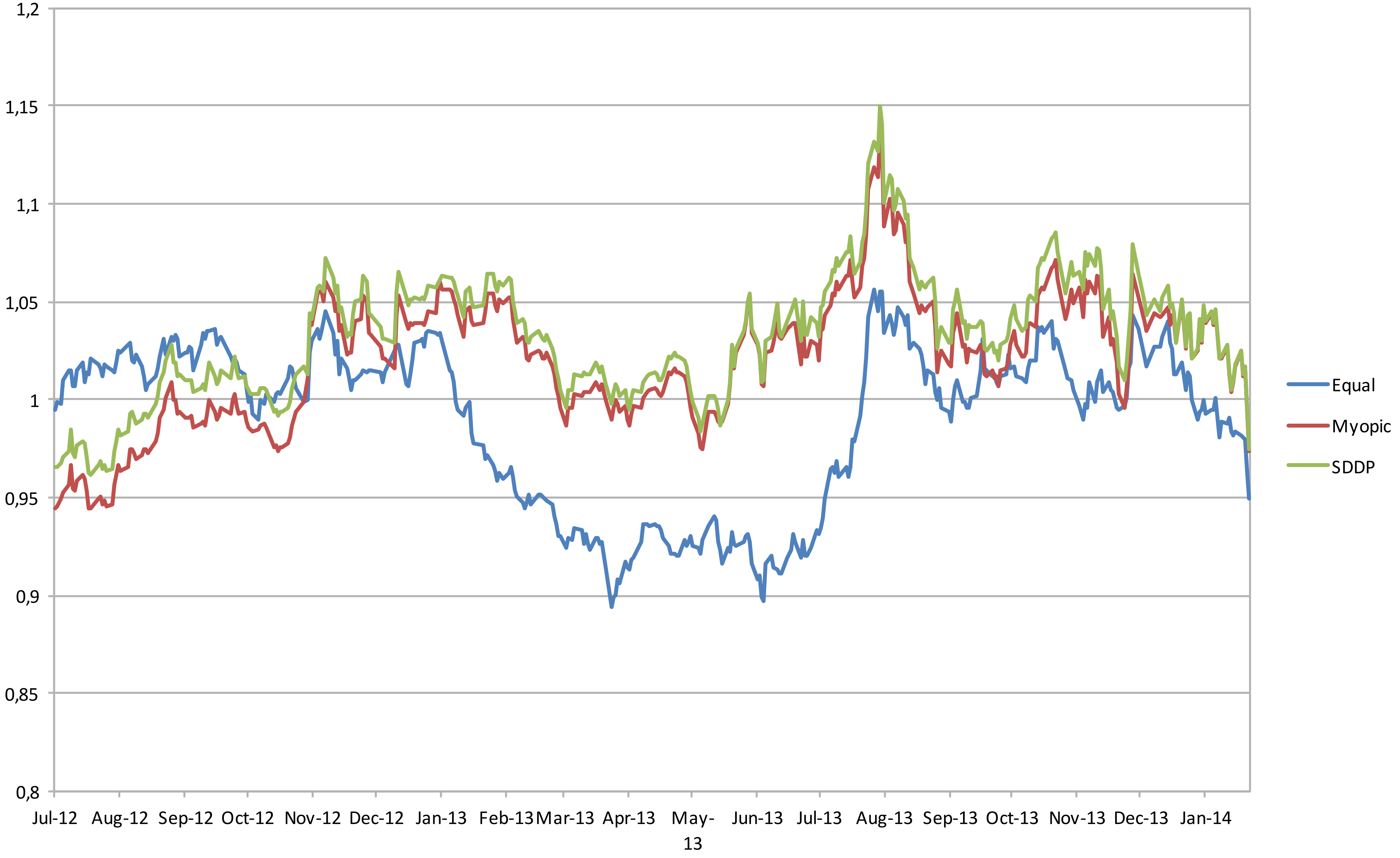


Case of study

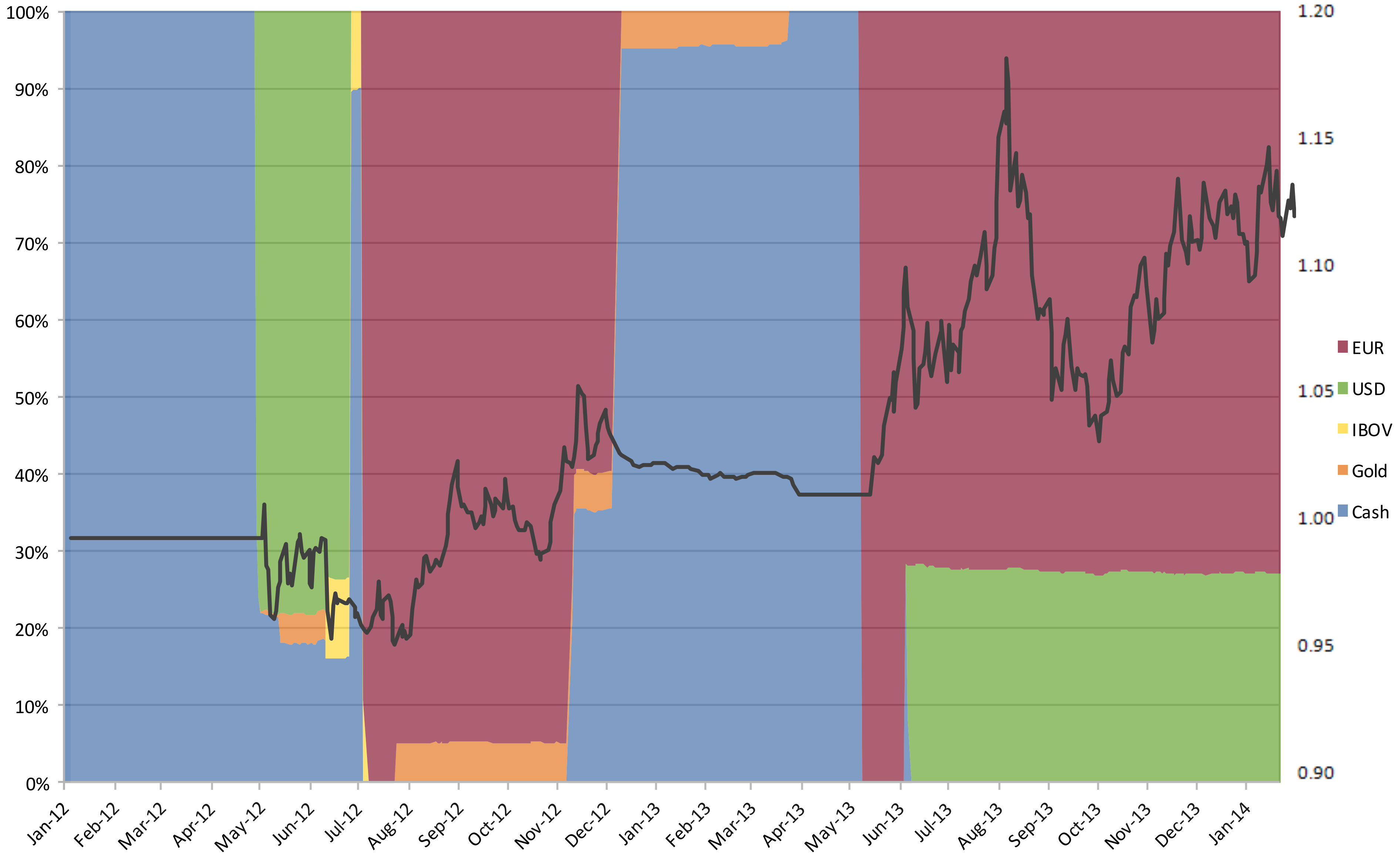
- Five types of investments: gold, IBOV, dollar and euro
- 4 states for HMM, 0.05% transactional costs, $\alpha = 0.9$ and $\gamma = 0.1$
- Uses 2 years with sliding window to evaluate the model, starting in 2008
- Simulate on out of sample data with 10 stages (days)
- Besides the cumulative performance we also evaluate the performance for 6 consecutive months

Cumulative portfolio return

Portfolio trailing return



Portfolio allocation



Conclusions

We presented a risk-constrained dynamic asset allocation model with transaction cost and time dependence assuming a Markov Model for asset returns.

Main Contributions:

- Time consistent risk-constrained stochastic dynamic asset allocation model
  - Realistic: transaction costs and time dependence
  - Risk aversion: intuitive user defined loss limit
  - Computationally tractable: SDDP with Markovian policy

Thank you!!!

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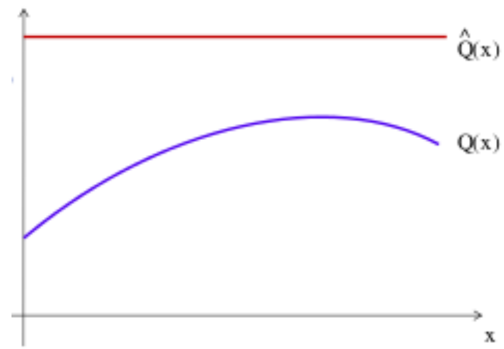
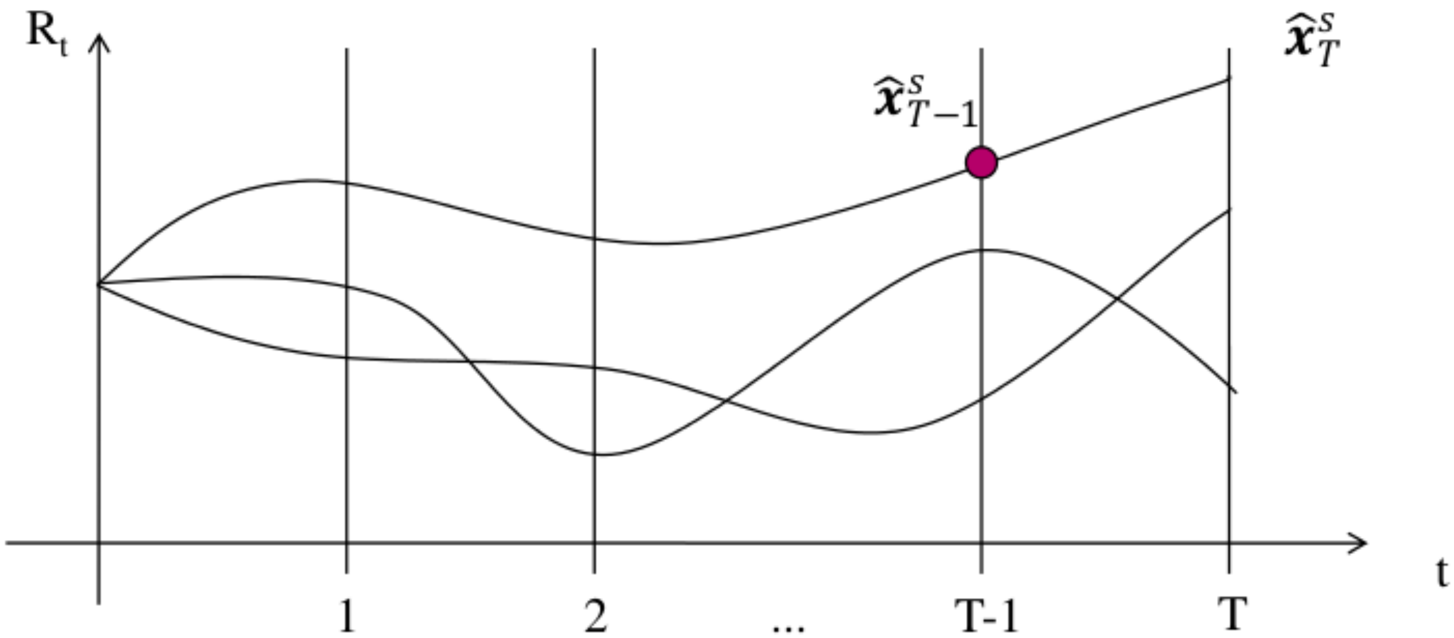
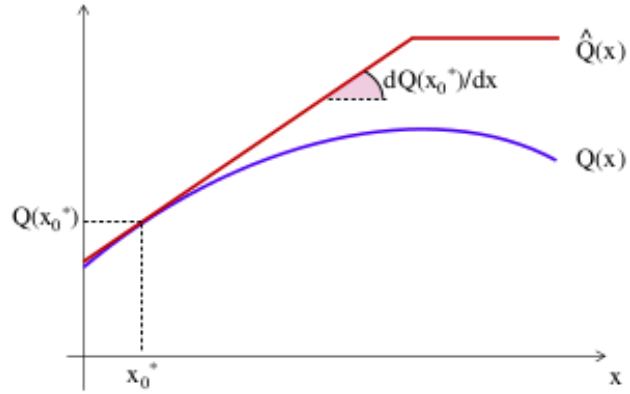
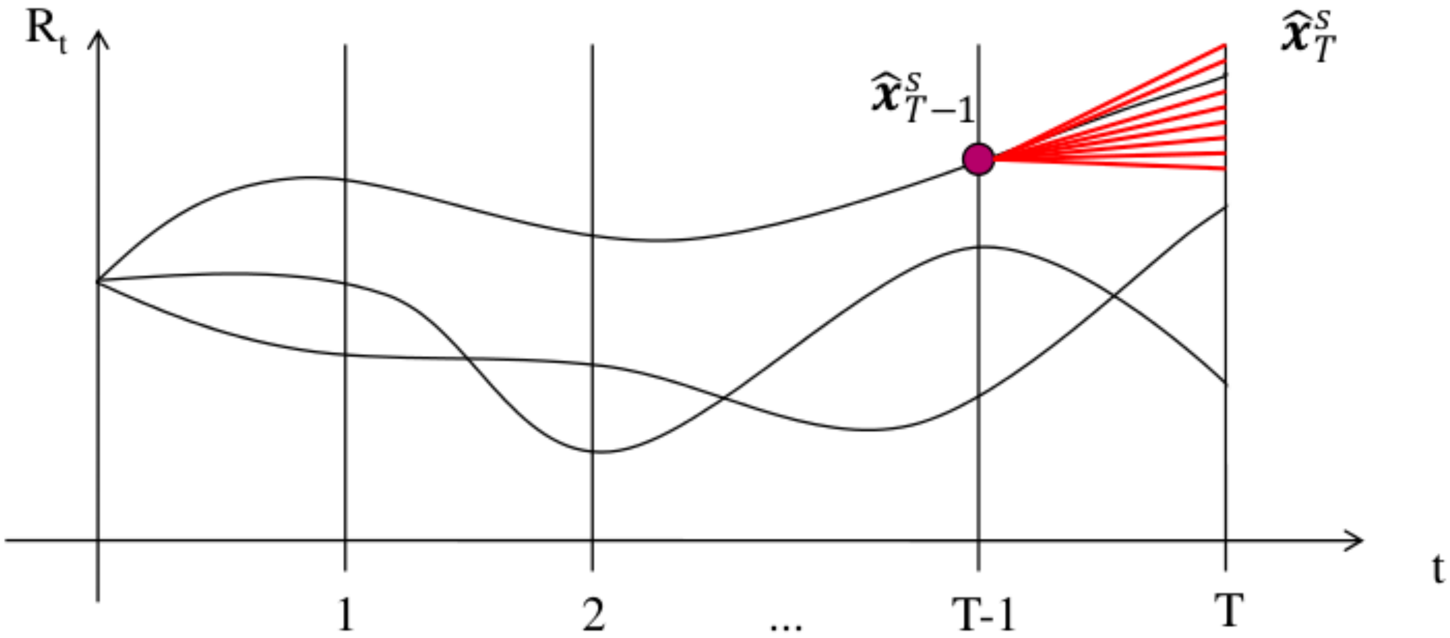
Stochastic Dynamic Dual Programming



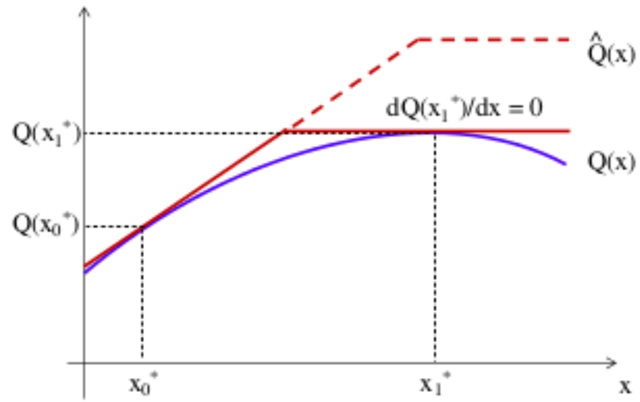


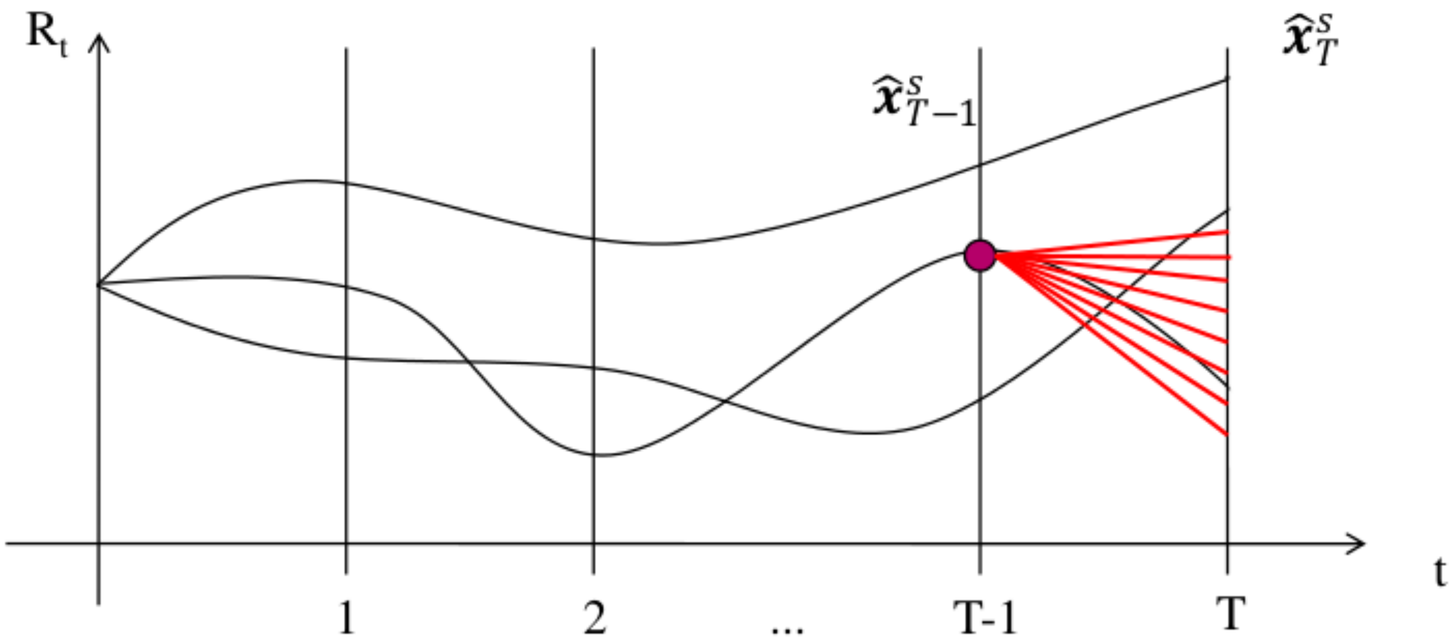
Stochastic Dynamic Dual Programming





Stochastic Dynamic Dual Programming





RHS dependence

$$Q_t(x_{t-1}, \xi_t) = \min_{x_t} \; c_t^\top \cdot x_t + \mathcal{Q}_{t+1}(x_t)$$
$$\text{s.t.} \;\; A_t \cdot x_t = -B_t x_{t-1} + b_t$$
$$x_t \geq 0$$

$$b_t = \phi b_{t-1} + \varepsilon_t$$

⇓

$$Q(x_{t-1}, b_{t-1}, \xi_t) = \min_{x_t} \quad c_t^\top \cdot x_t + \mathcal{Q}_{t+1}(x_t)$$
$$\text{s. t.} \quad A_t x_t = -B_t x_{t-1} + \phi_t b_{t-1} + \varepsilon_t$$
$$x_t \geq 0$$