On the modeling and solution strategy of the Long Term Hydrothermal Scheduling

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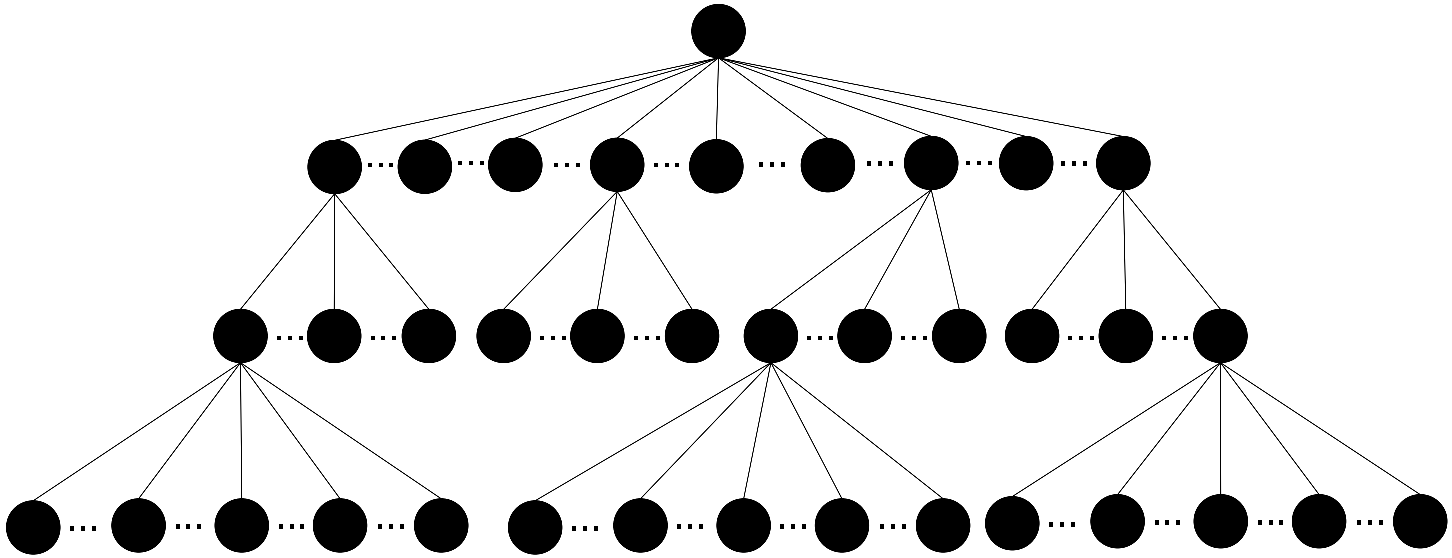
Analysis and Applications of Stochastic Systems – AASS – 2016

Large Scale Optimization

- Several **applications are Large Scale**
- The strategy is to decompose in **smaller subproblems**
- We keep a connection between the subproblems that is **improved iteratively**
- There are several algorithms for **Stochastic Programming**
  - We are interested in **SDDP**

Large Scale Optimization

- Scenario Tree



Large Scale Optimization

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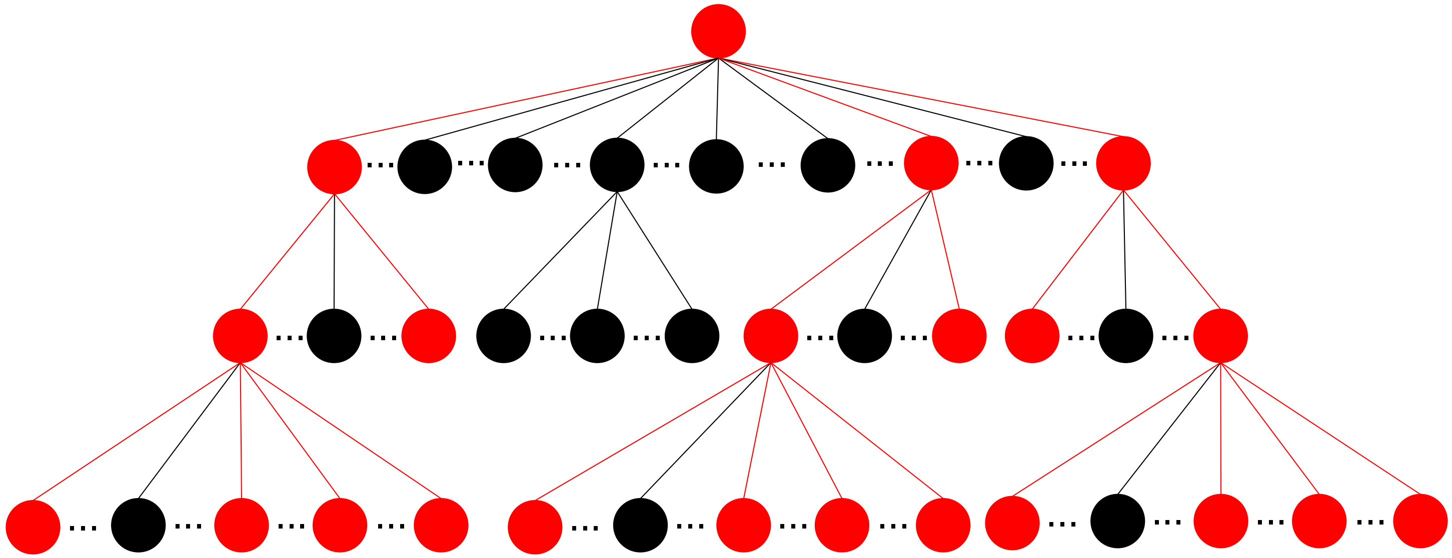
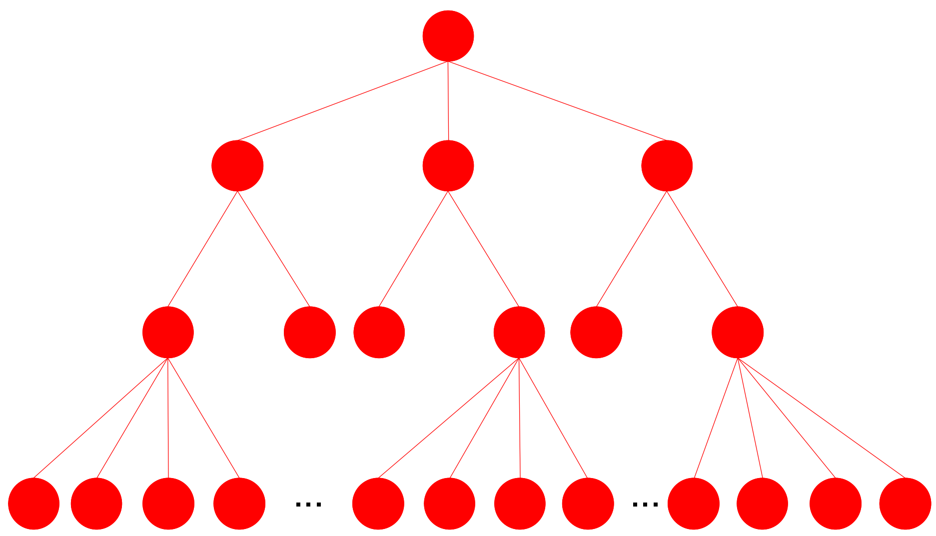


  - Sampling → Monte Carlo (MC)

Large Scale Optimization

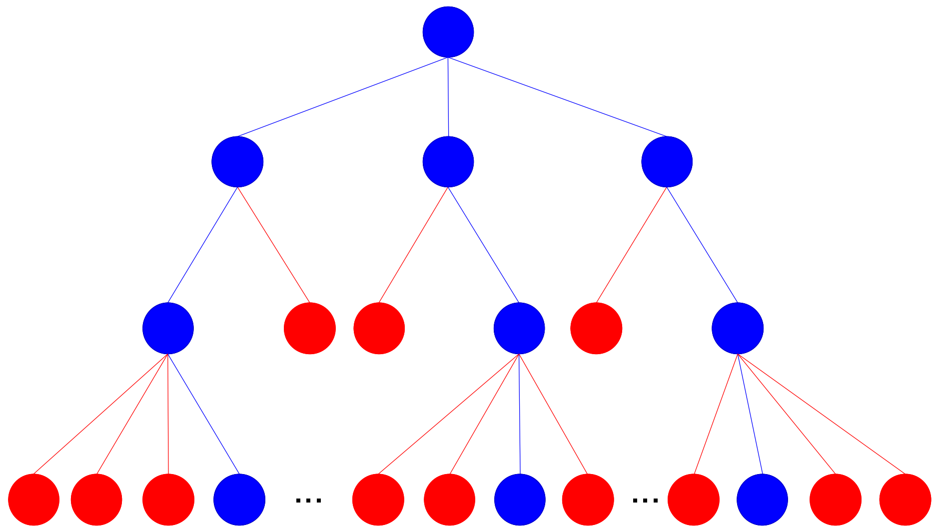
- It is not possible to solve the whole problem
  - Due to the number of stages



SDDP

- An alternative is to use a sampling algorithm

- Stochastic Dual Dynamic Programming - SDDP
  - Sample scenarios → Monte Carlo



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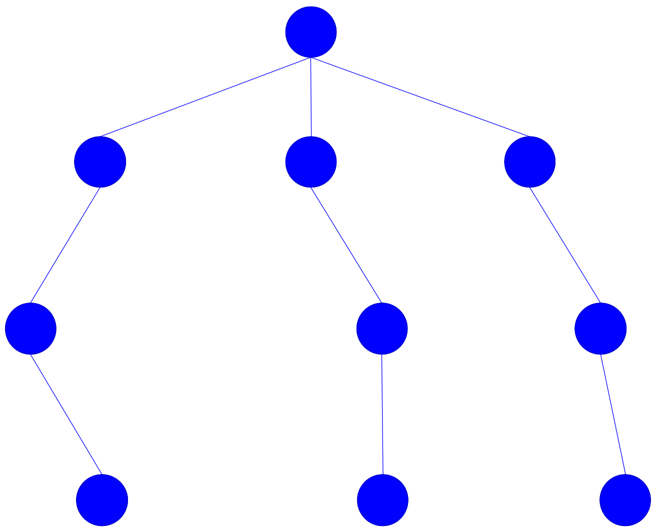
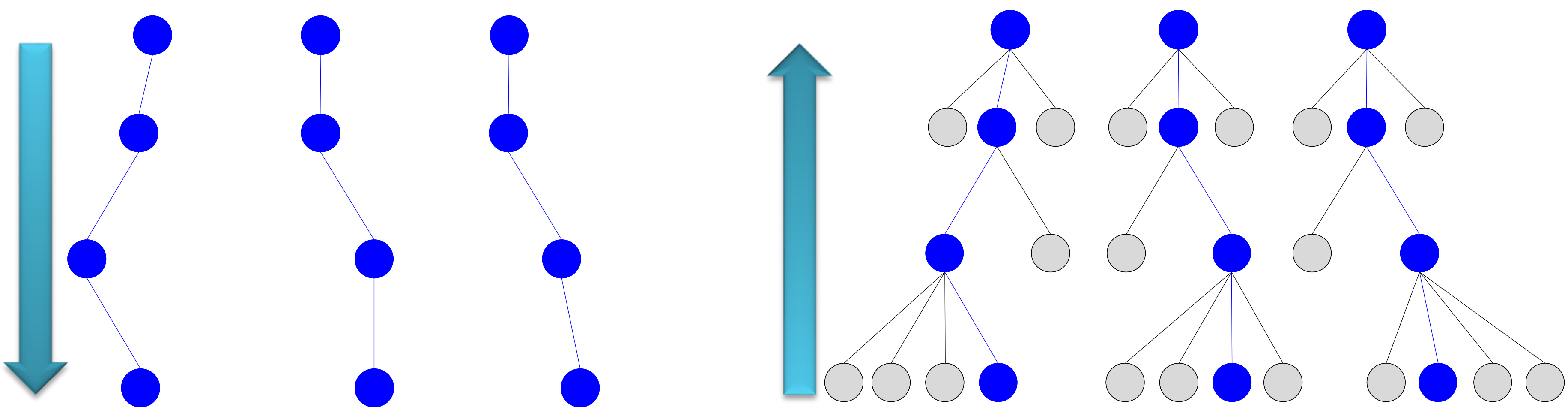
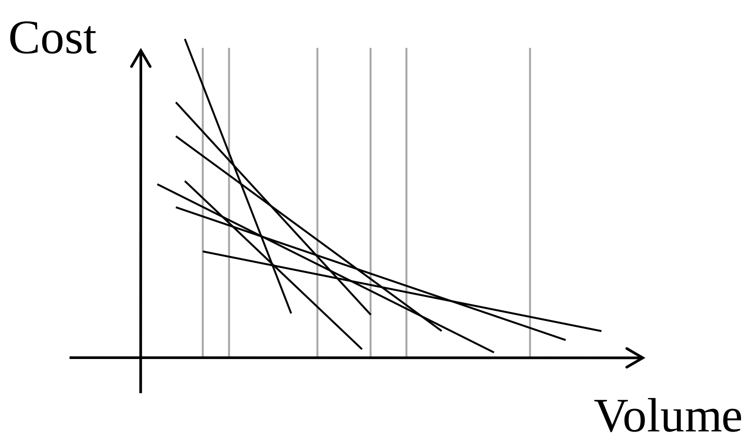


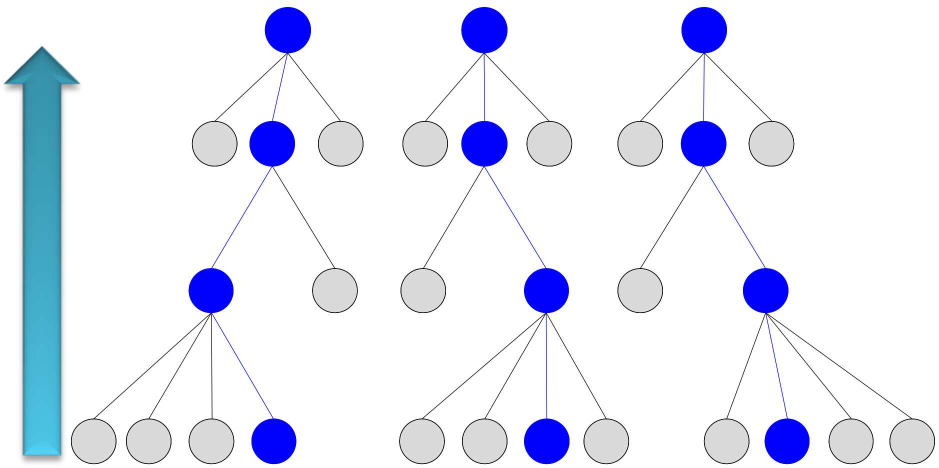
SDDP

- SDDP has two main steps:
  - *Forward*
  - *Backward*



- In the backward we add information to our policy
  - Future Cost Function (FCF)
  - Benders Cuts or, simply, Cuts





SDDP - Summary

- Breaks the problem into **node and scenarios**
- **Sampling scenarios** allow building a policy for multi-stage scenario trees
- Connection is through a **Cost-to-go Function**
  - Built as a piecewise linear approximation **over the state space**
- **Requires longer** if
  - the number of stages grow
  - the number of random variables grow
  - the number of state variables grow

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  - the number of state variables grow

What if we grow the random variables and the state space?

Recent advances in Sampling Methods...

- ...such as,
  - Different **tree traversing** strategies
  - **Cut selection** to reduce LP size
  - **Risk aversion** methods
  - Scenario **reduction techniques** and **variance reduction** sampling
  - New strategies for **parallelization**
  - Etc...

- ... Led us to believe that it is possible to find **a solution/policy**

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  - Etc...

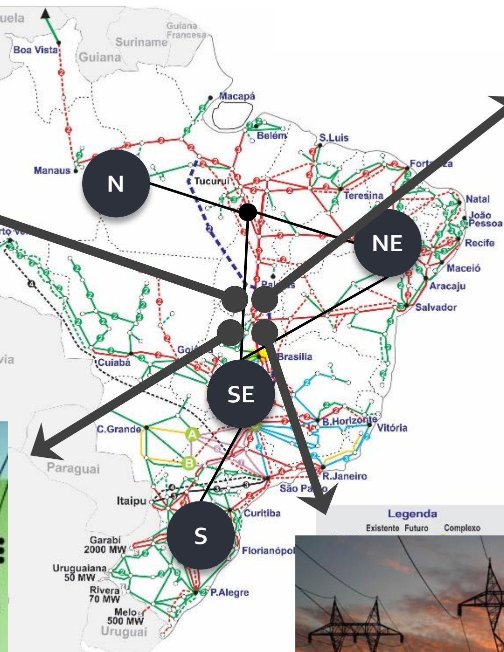
But, how good is it? Compared to what?

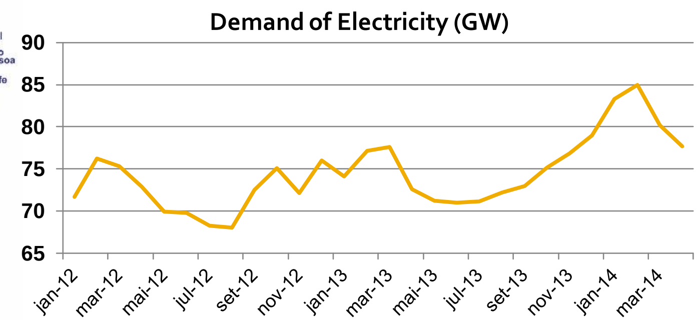
- ... Led us to believe that it is possible to find **a solution/policy**

Brazilian Electric System

➢ Main characteristics (Dec. 2014)

194
Hydro Plants | Storage Capacity
Run of River | **86.7 GW**
Installed





1.774
Thermal Units | Nuclear
Coal
Natural Gas
Diesel
Biomass | **39.7 GW**
Installed

approx.
110.000 km

Brazilian Planning Chain

Long Term:
- 10 years planning horizon
- Simplified Modeling
- Inflows are uncertain

Medium Term:
- 2 to 6 months horizon
- More detailed modeling
- Inflows are uncertain

Short Term:
- 48 hours to 1 week horizon
- Extremely detailed modeling
- Inflows are assumed known

Stochastic
Simplified Modeling

Deterministic
Detailed Modeling

Hydro Modeling

Hydro Modeling

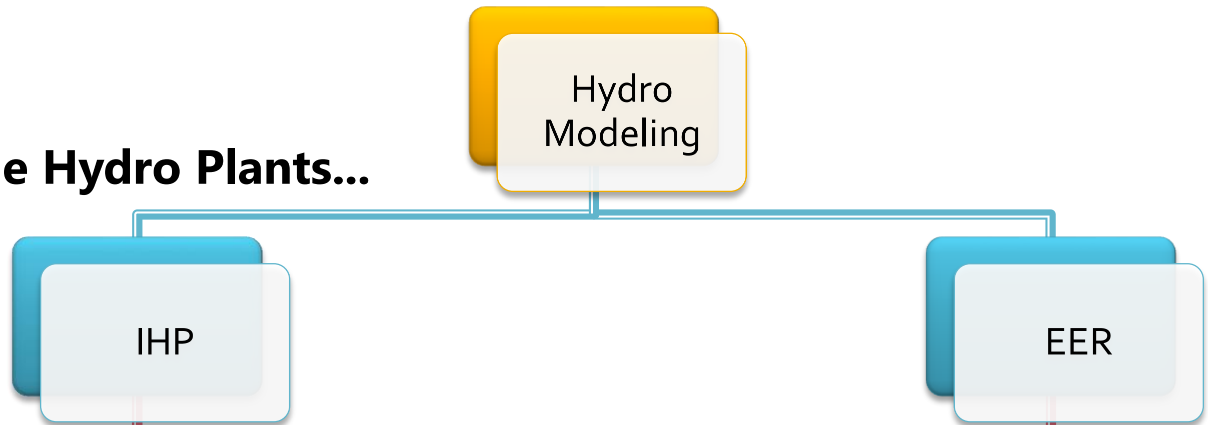
How to model the Hydro Plants...

How to model the Random Variables...

How to model the Inflow Process...

Hydro Modeling

How to model the Hydro Plants...



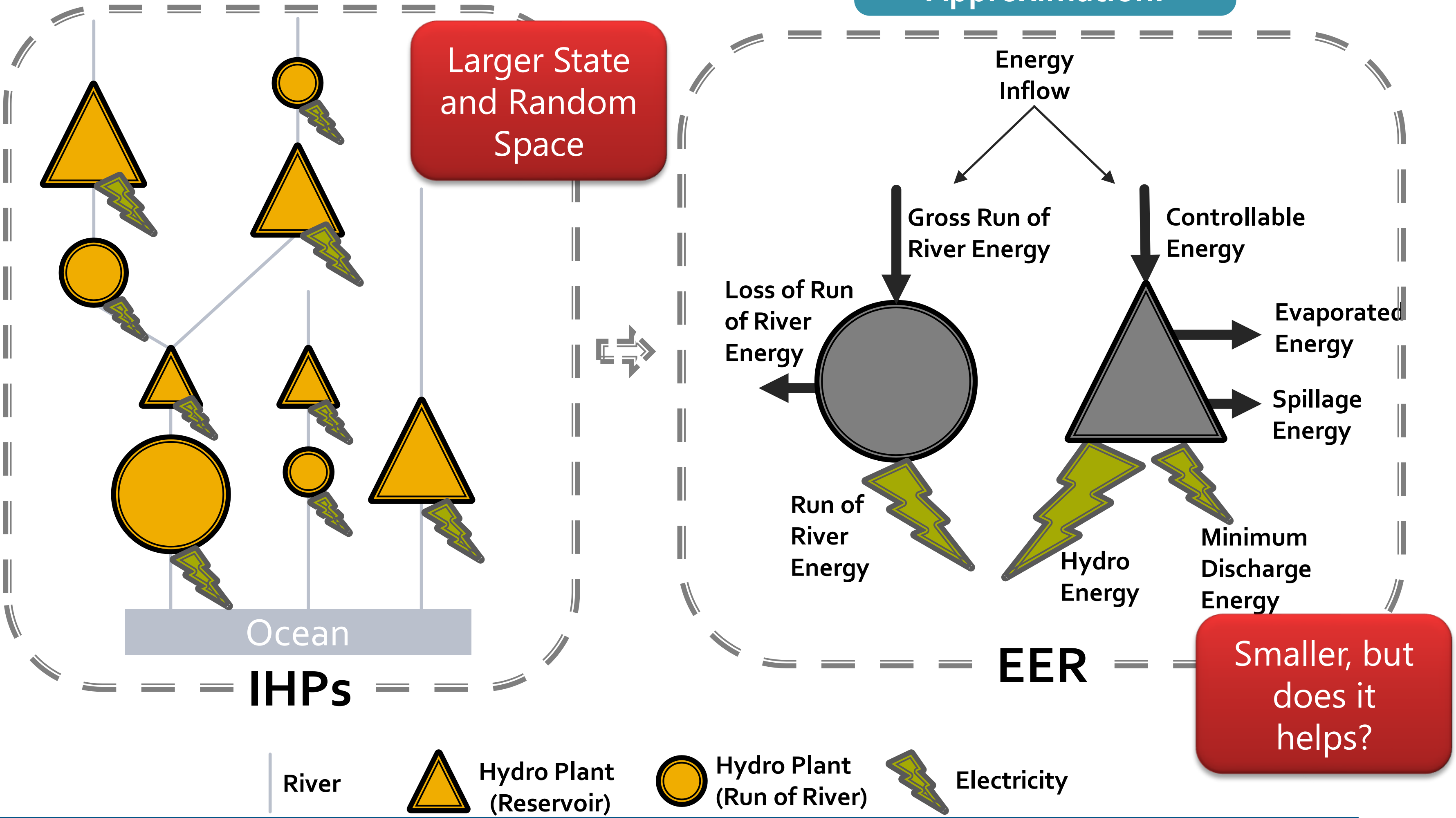
How to model the Random Variables...

How to model the Inflow Process...

Individual and EERs

- Energy Equivalent Reservoirs (EERs)

Optimistic Approximation!

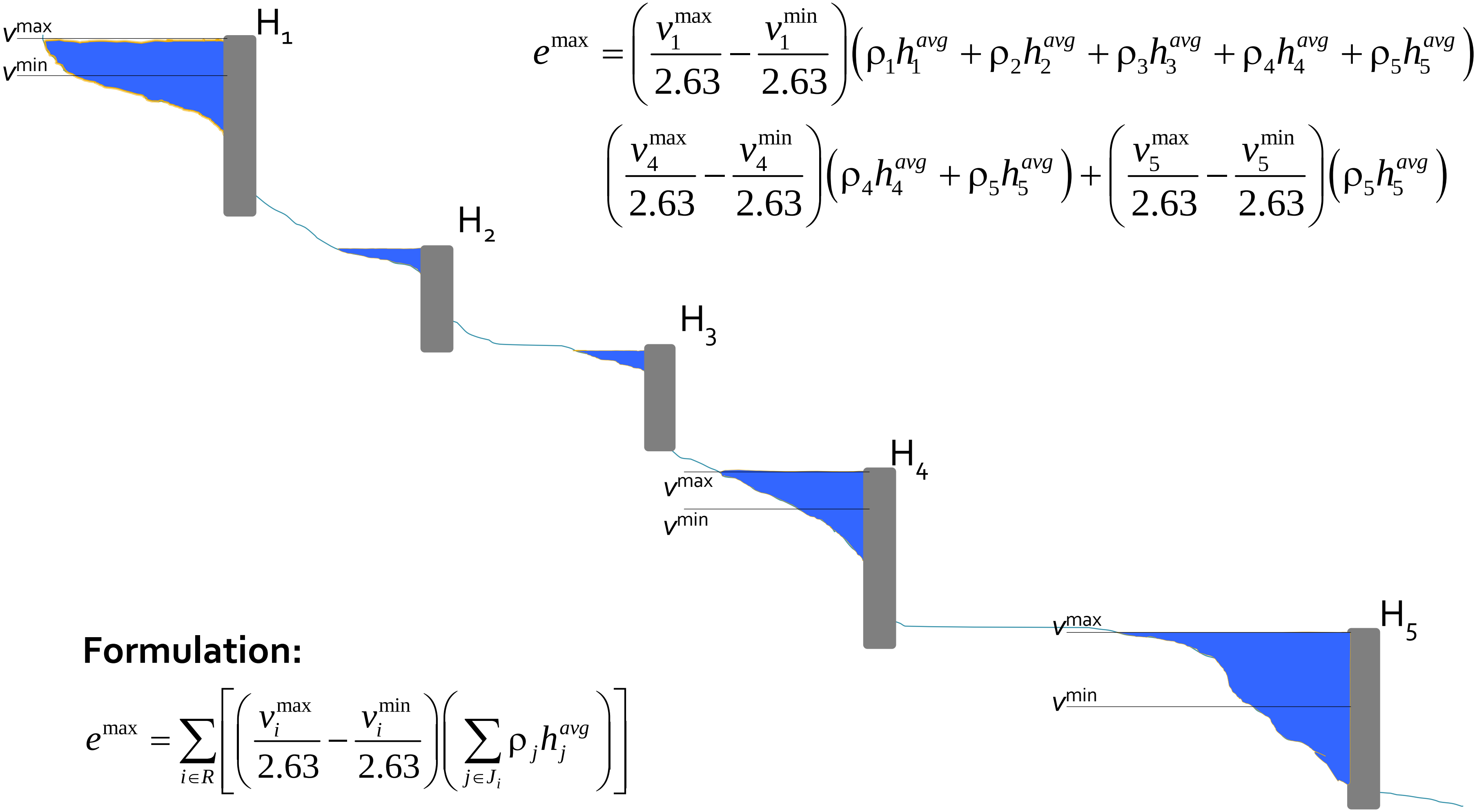


Energy Equivalent Reservoirs (EERs)

List of Attributes

- **Maximum Storage Energy**
- Minimum Storage Energy
- Natural Energy Inflow
- Controllable Energy Inflow and Run-of-River Energy Inflow
- Loss of Run-Of-River Energy Inflow
- Correction Factor of Controllable Energy Inflow
- Minimum Discharge Energy
- Evaporated Energy
- Maximum Energy Generation
- Small Hydroplants Energy
- Production Before Full Commitment
- Fulfilling Minimum Stored Energy
- Controllable Inflow Deviation Energy
- Run-of-River Inflow Deviation Energy

Energy Equivalent Reservoirs (EERs)
Maximum Storage Energy



$$e^{\max} = \left(\frac{v_1^{\max}}{2.63} - \frac{v_1^{\min}}{2.63}\right)\left(\rho_1 h_1^{avg} + \rho_2 h_2^{avg} + \rho_3 h_3^{avg} + \rho_4 h_4^{avg} + \rho_5 h_5^{avg}\right) + \left(\frac{v_4^{\max}}{2.63} - \frac{v_4^{\min}}{2.63}\right)\left(\rho_4 h_4^{avg} + \rho_5 h_5^{avg}\right) + \left(\frac{v_5^{\max}}{2.63} - \frac{v_5^{\min}}{2.63}\right)\left(\rho_5 h_5^{avg}\right)$$

Formulation:

$$e^{\max} = \sum_{i \in R}\left[\left(\frac{v_i^{\max}}{2.63} - \frac{v_i^{\min}}{2.63}\right)\left(\sum_{j \in J_i} \rho_j h_j^{avg}\right)\right]$$

Larger and Smaller... in numbers

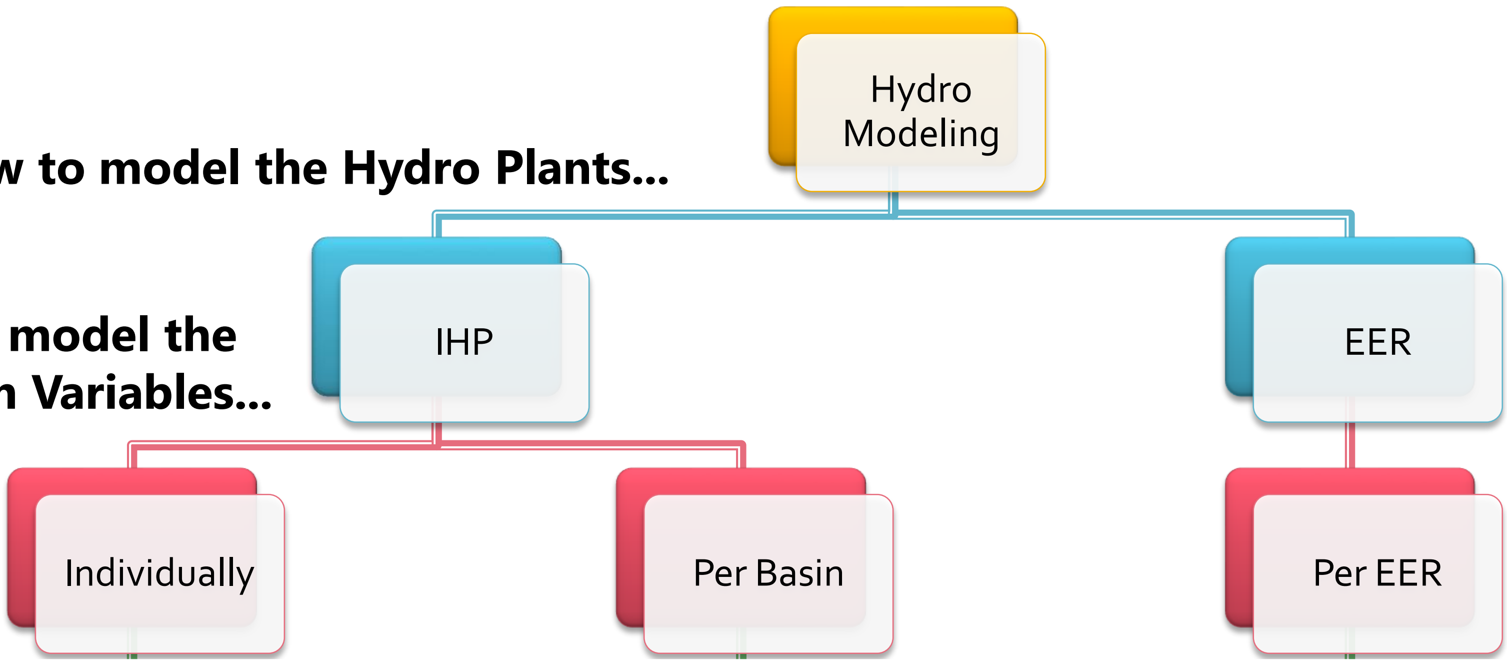
- When considering inflows **per Plant** and PAR(1)

	EER	IHP	Difference (%)
Random Variables	4	151	3775
State Variables	8	218	2725
Decision Variables	501	1236	247
Constraints*	16	163	1019
* No cuts taken into account.			

Hydro Modeling

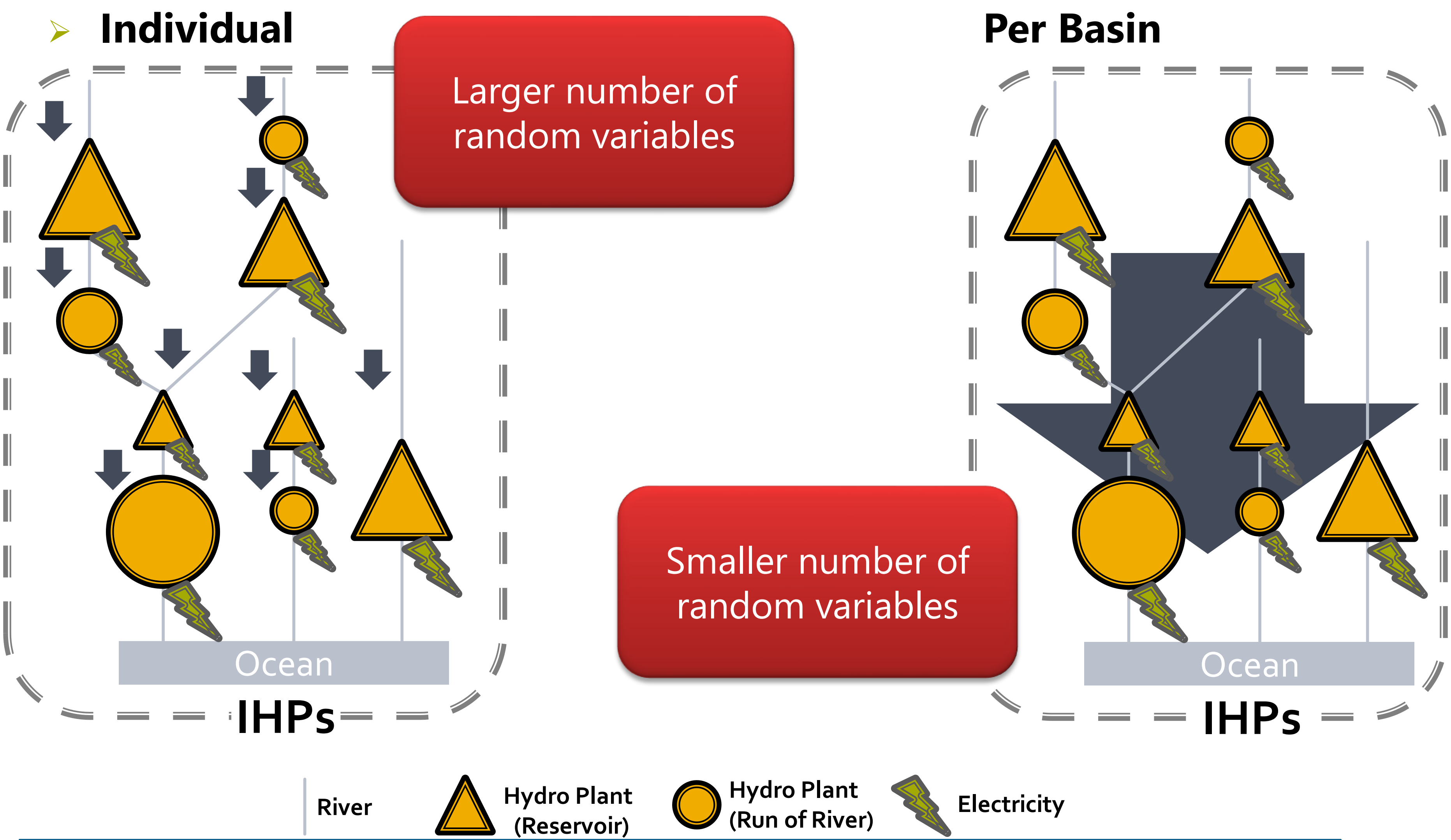
How to model the Hydro Plants...

How to model the Random Variables...



How to model the Inflow Process...

Random Variable modeling



Larger and Smaller… in numbers

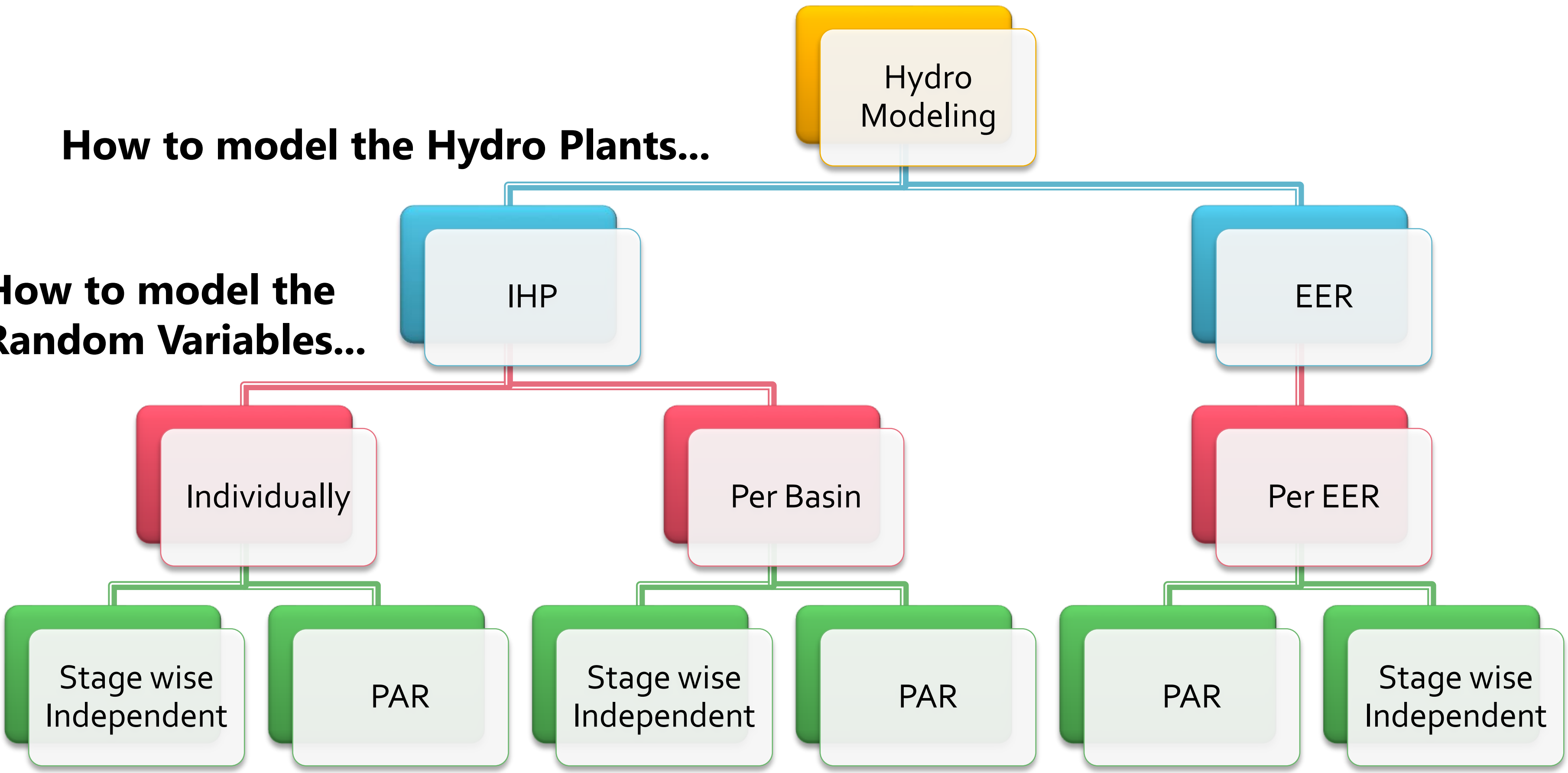
- But, what if we consider inflows **per Basin**…

	IHP Hydro	IHP Basin	Difference (%)
Random Variables	151	16	89
State Variables	218	83	62
Decision Variables	1236	1236	0
Constraints*	163	163	0
*** No cuts taken into account.**			

Hydro Modeling

How to model the Hydro Plants...

How to model the Random Variables...



How to model the Inflow Process...

Inflow scenario generation model

- Periodic Auto Regressive model – PAR(*p*)
  - Order identification using ACF/PACF
  - Auto Regressive coefficients are computed using ACF
  - We fit a 3-Parameter LogNormal to the historical residuals

$$\left(\frac{y_{rt}-\mu_{rm}}{\sigma_{rm}}\right)=\phi_{t1}\left(\frac{y_{r,t-1}-\mu_{r,m-1}}{\sigma_{r,m-1}}\right)+\ldots+\phi_{rtp}\left(\frac{y_{r,t-p}-\mu_{r,m-p}}{\sigma_{r,m-p}}\right)+\kappa_{rt}^{\omega_t}$$

- Independent model
  - We fit a 3-Parameter LogNormal to the historical inflows

$$y=\exp(\xi\sigma_x+\mu_x)+\Lambda.$$

Hydro Modeling

How to model the Hydro Plants...

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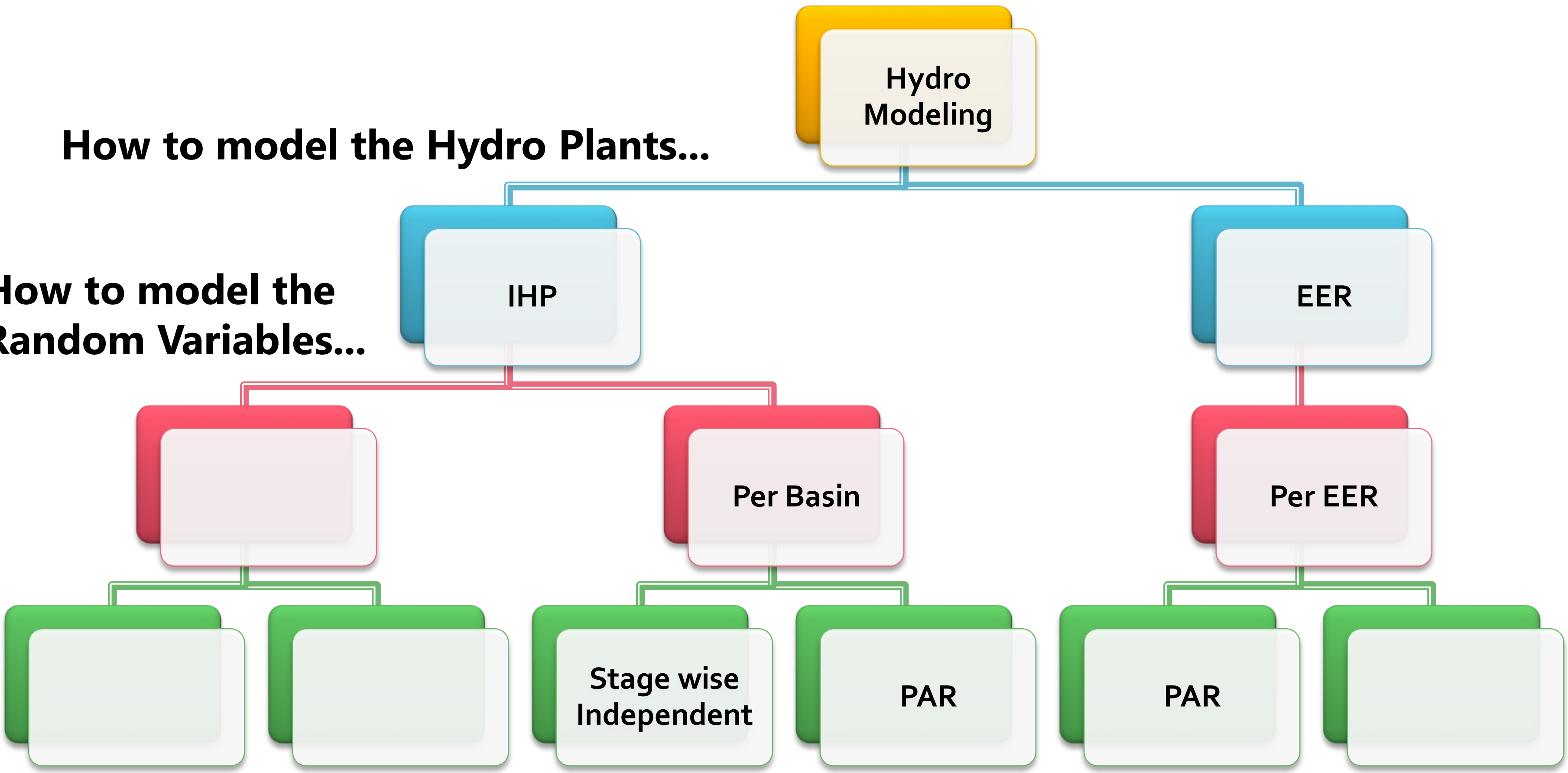


Computational Experiment

- Base case
  - IHP - Random variables **per Basin** and **PAR(p)**
- Comparison
  - EER - Random variables **per EER** and **PAR(p)**
  - IHP - Random variables **per Basin** and **Stagewise Independent**
- Configuration
  - SDDP algorithm
  - 30% initial condition (volumes)
  - 5-year horizon with monthly stages
  - 50 realizations per stage
  - 30 iterations with 200 scenarios per iteration
  - 10 threads (20 scenarios per thread in each iteration)

Computational Experiment

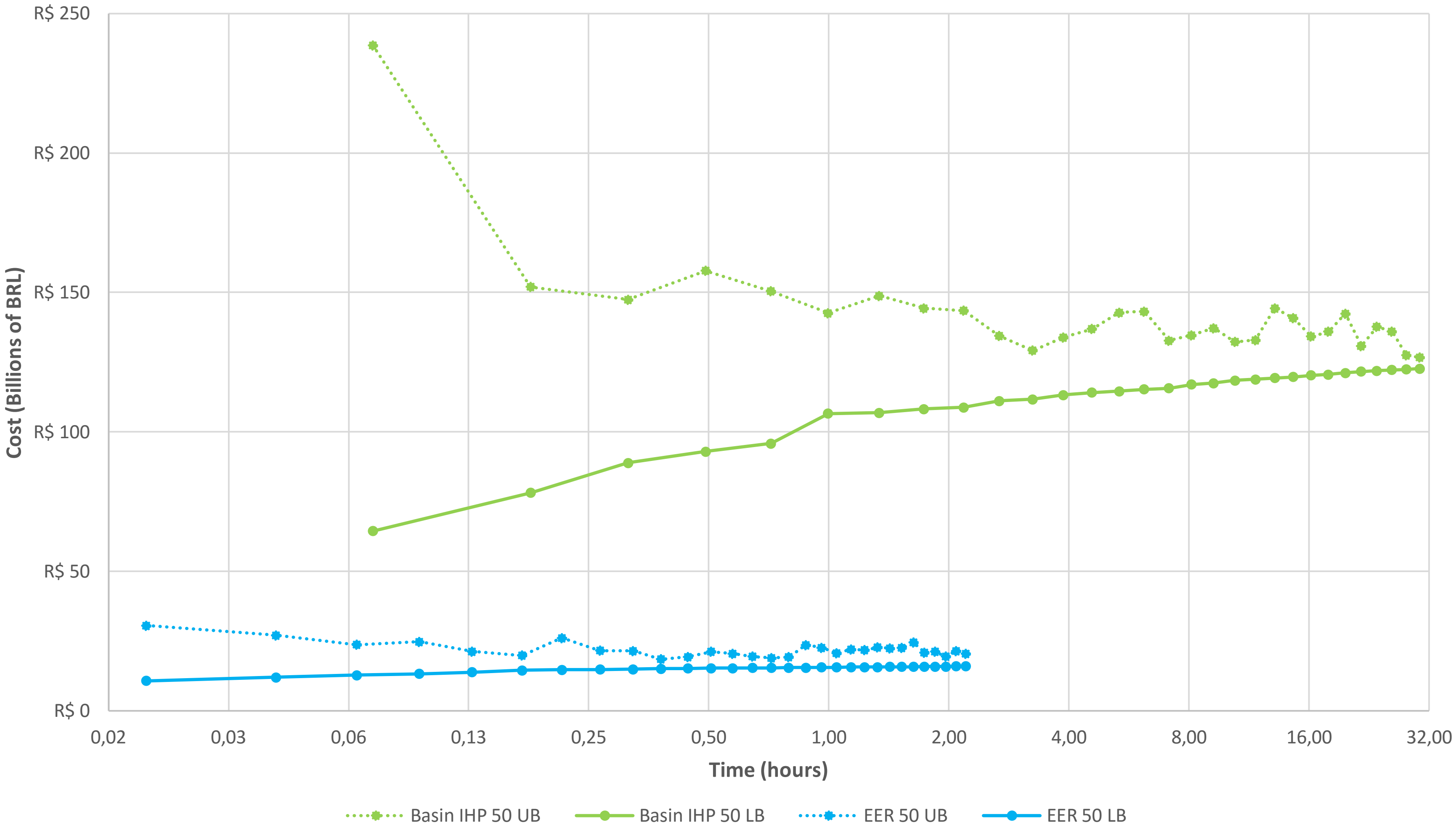
- **Cost-to-go Function** is computed for the following conditions:
  - IHP - Random variables **per Basin** and **PAR(p)**
  - EER - Random variables **per EER** and **PAR(p)**
  - IHP - Random variables **per Basin** and **Stagewise Independent**

- But all **simulations** are based on
  - IHP - Random variables **per Hydro Plant** and **PAR(p)**

2000 Outsample Scenarios

77 Historical Series

Lower and Upper Bounds



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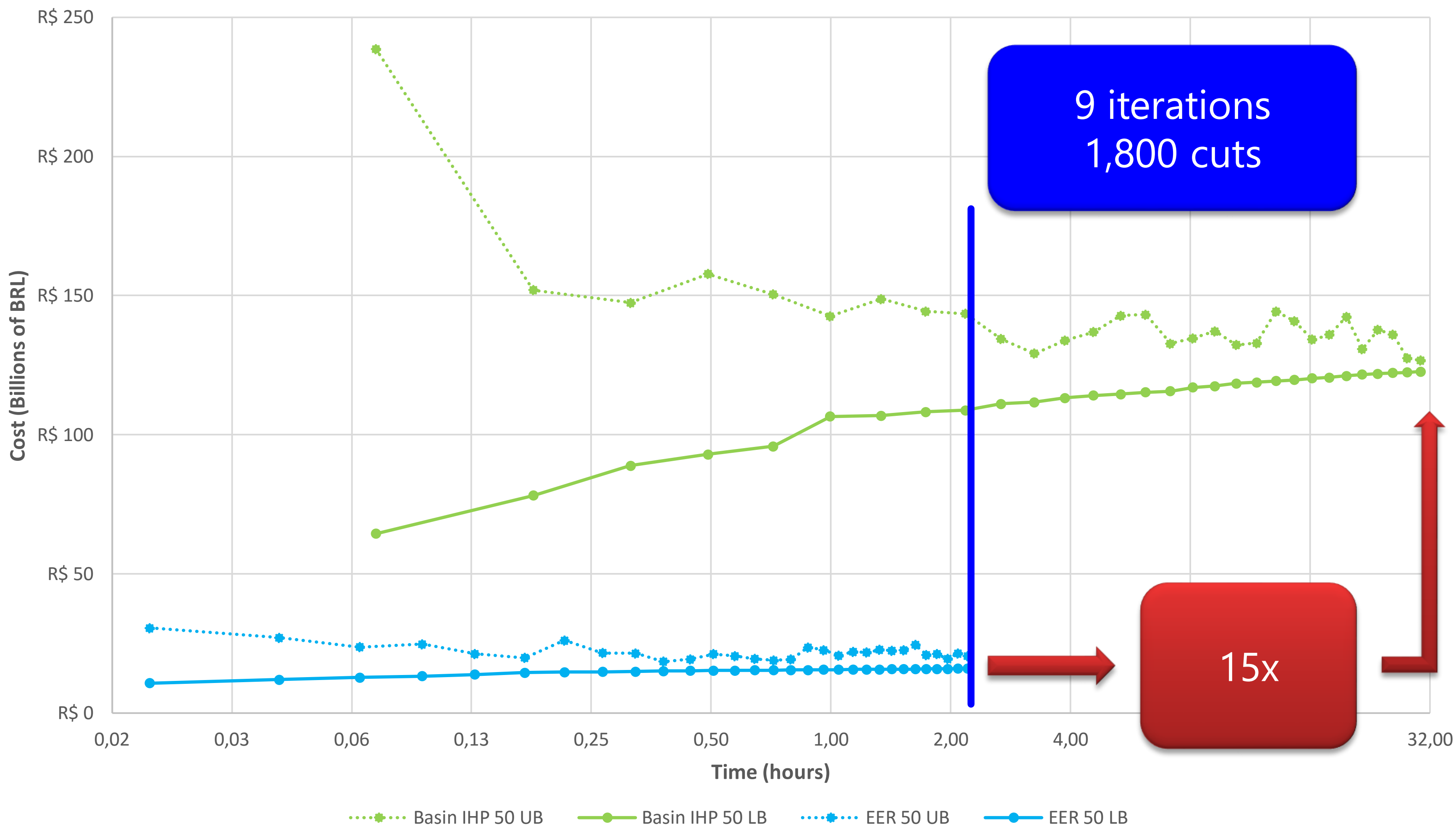
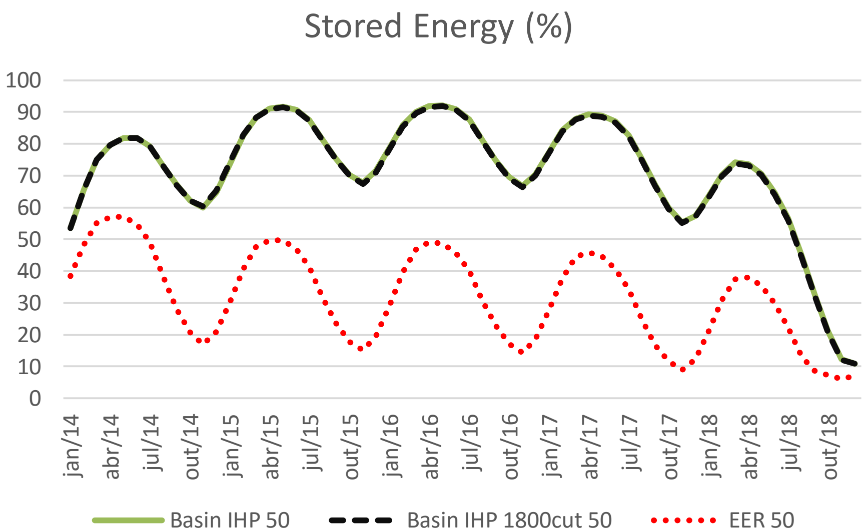
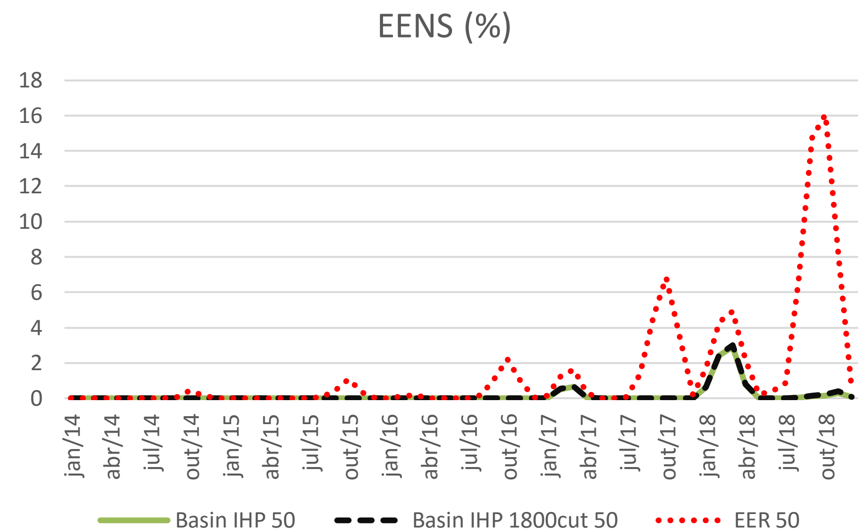


Comparing IHP and EER

Simulation using 2000 outsample considering Individual Inflows





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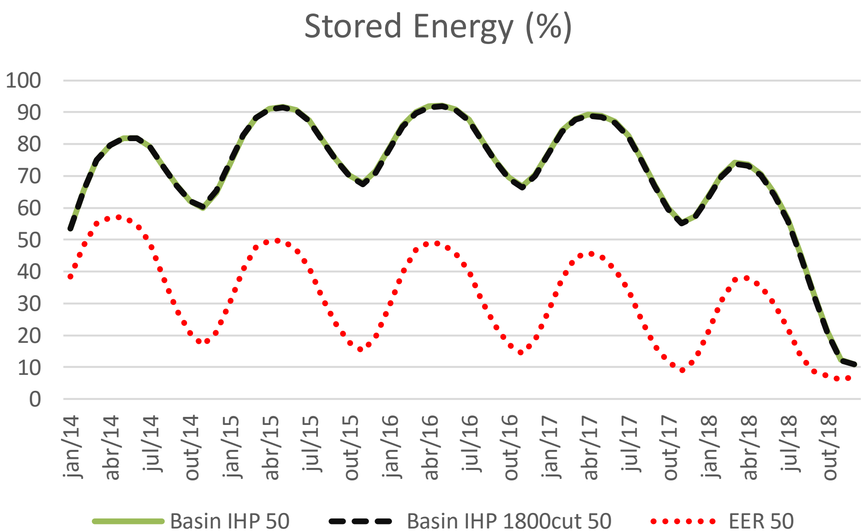
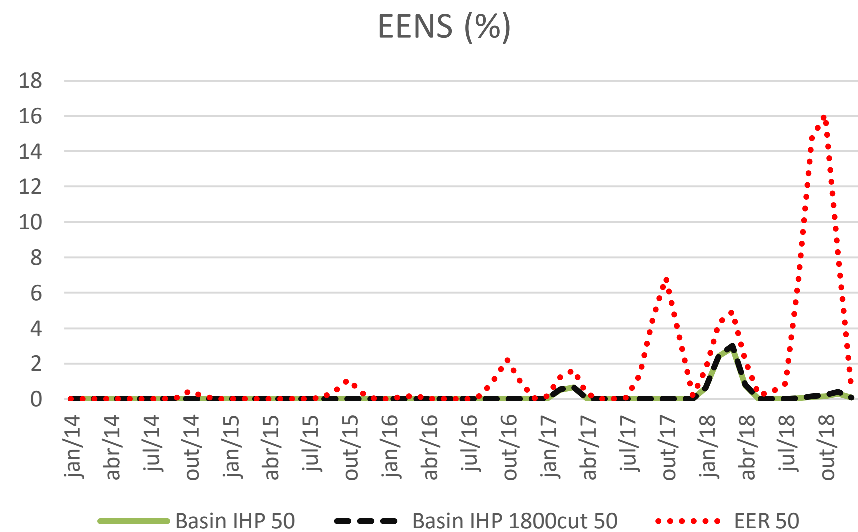




Expected Operation Cost (Billions of BRL – R$)

3 Years

EER 30.03

IHP 34.79

IHP 1800 35.04

14%

1%

5 Years

EER 130.40

IHP 60.59

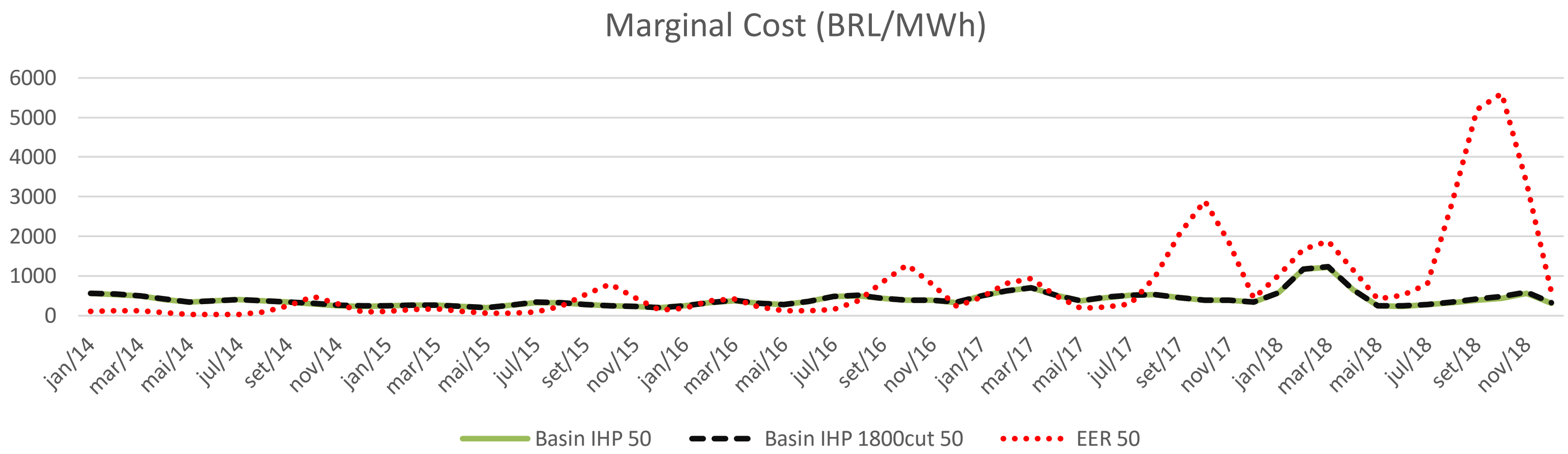
IHP 1800 61.31

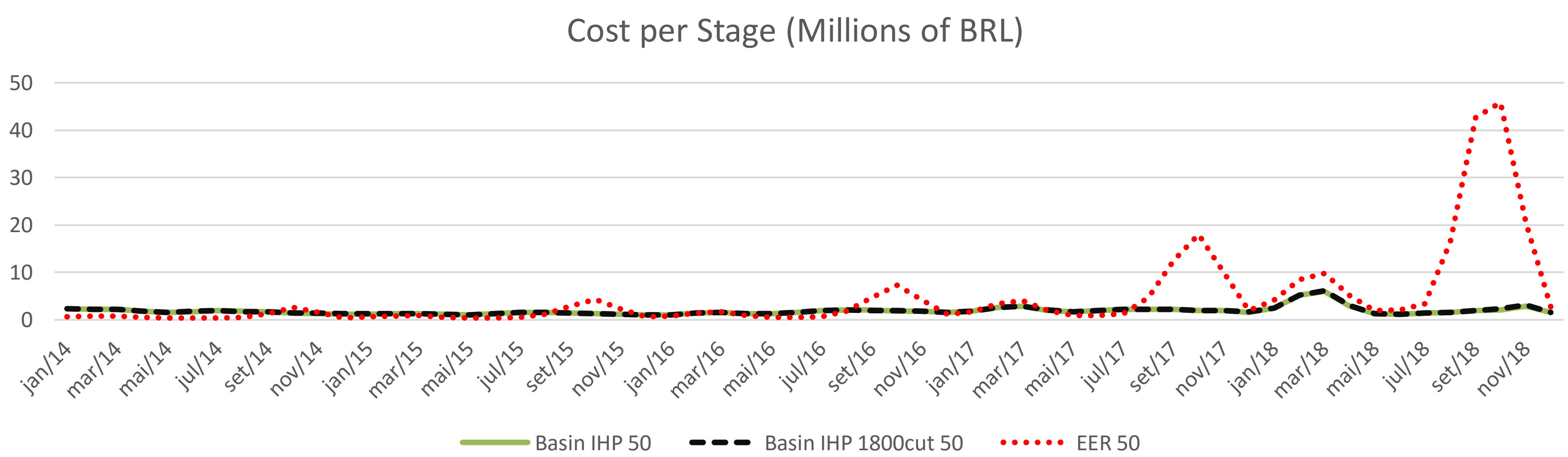
54%

1%

Comparing IHP and EER

Simulation using 2000 outsample considering Individual Inflows





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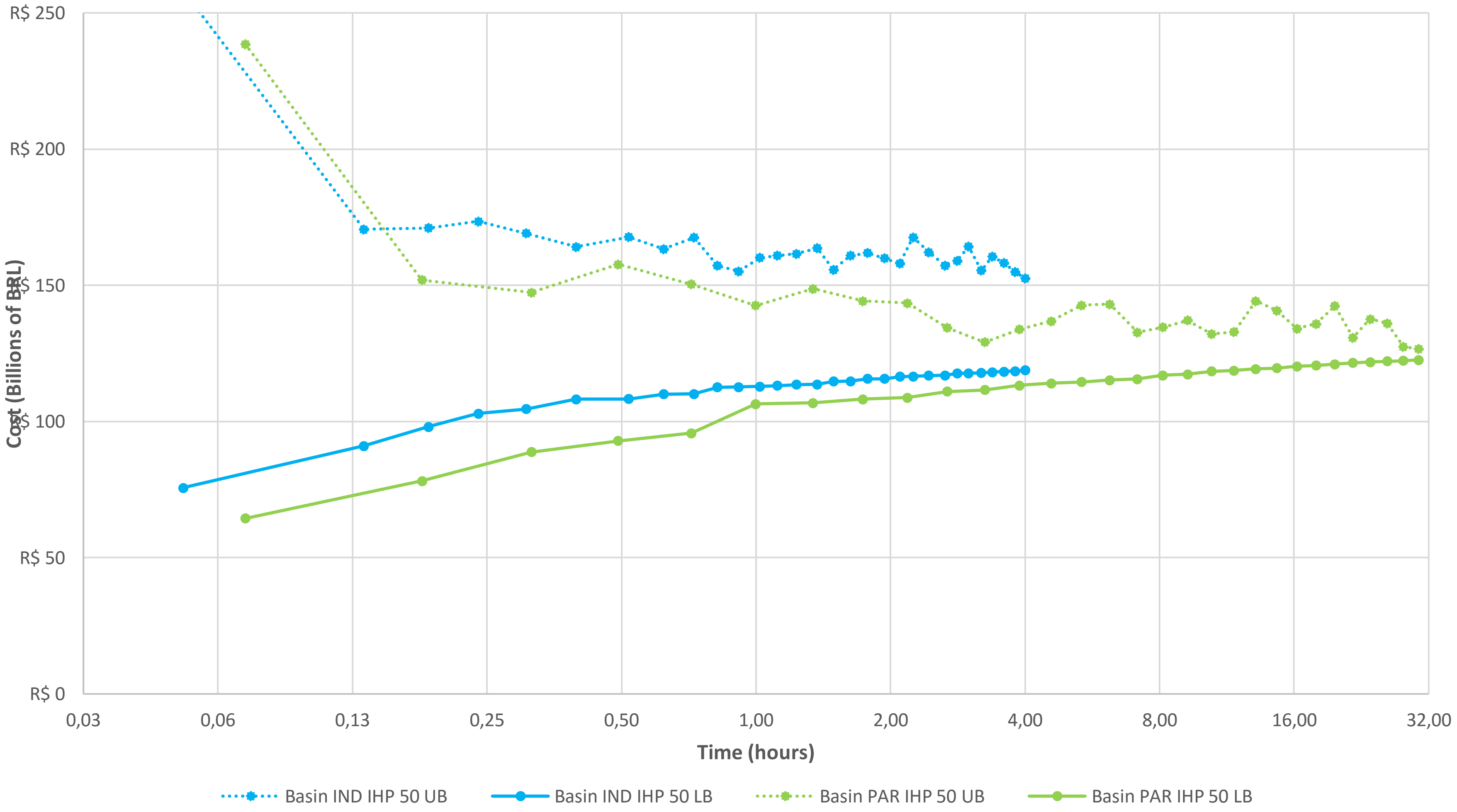

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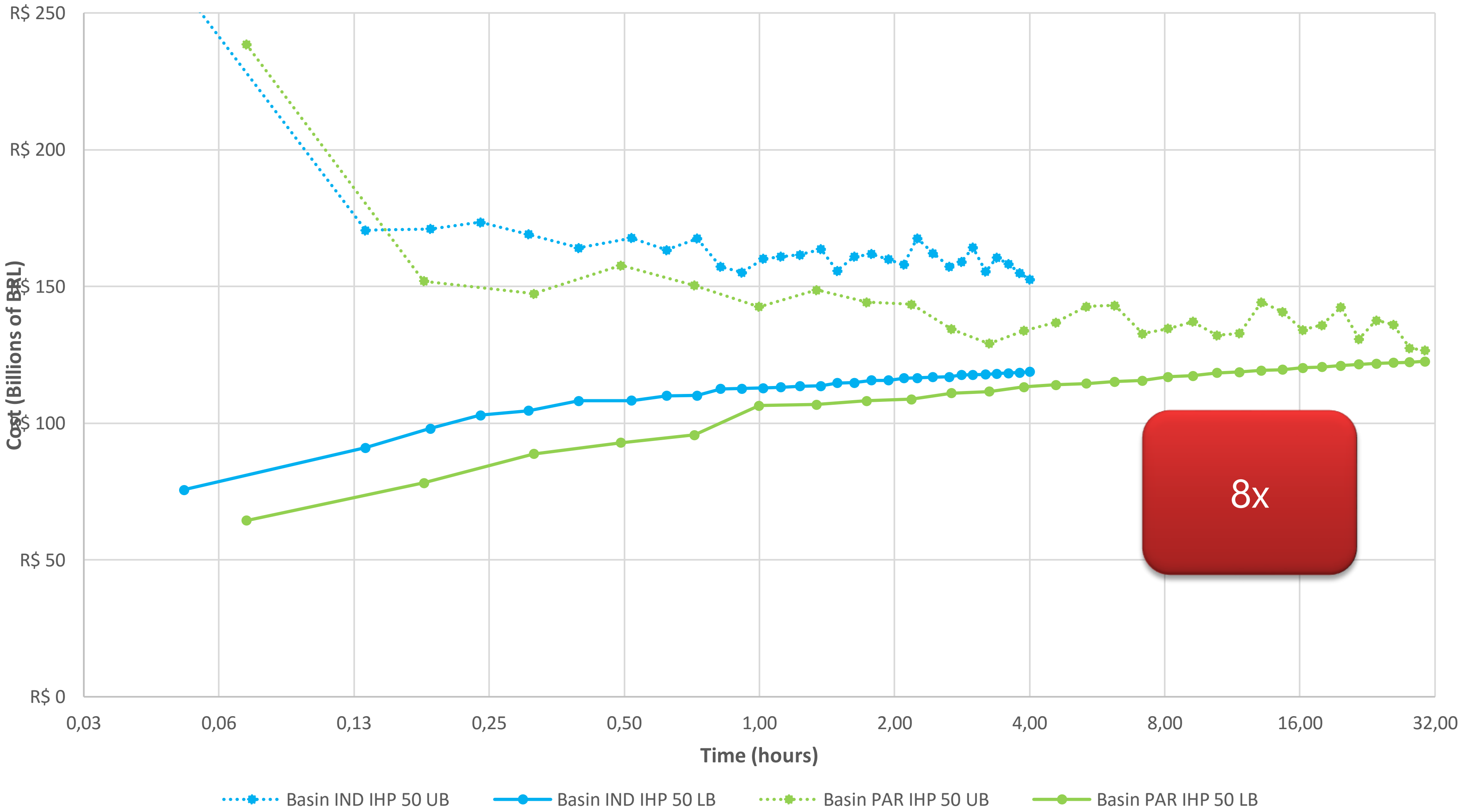
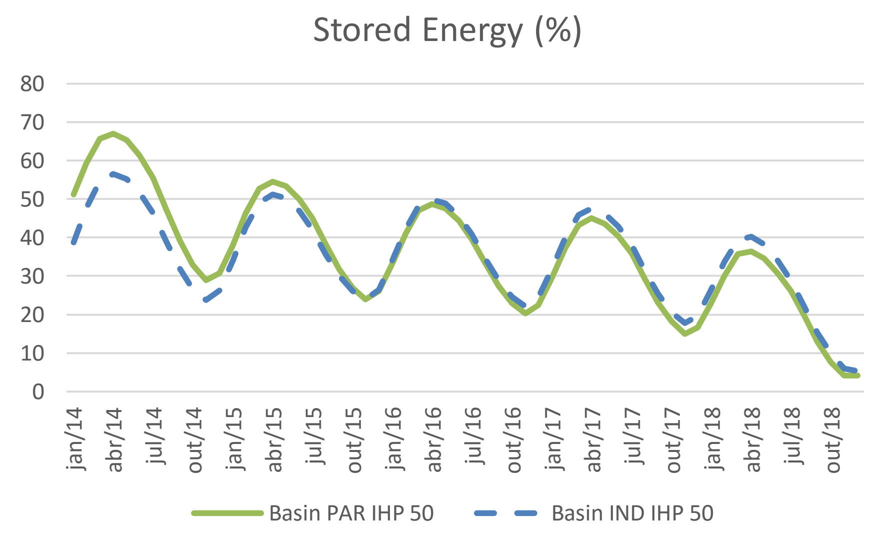
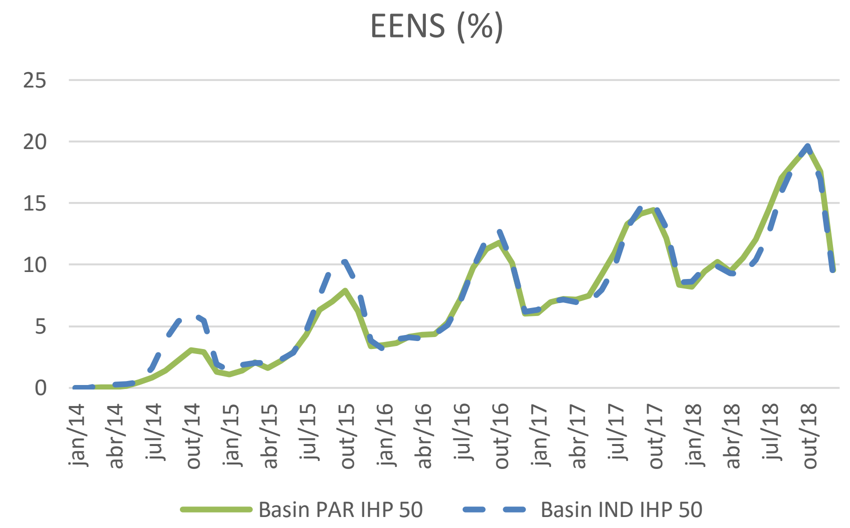


Comparing Independence and PAR

Simulation using 77 historical series of 5 years





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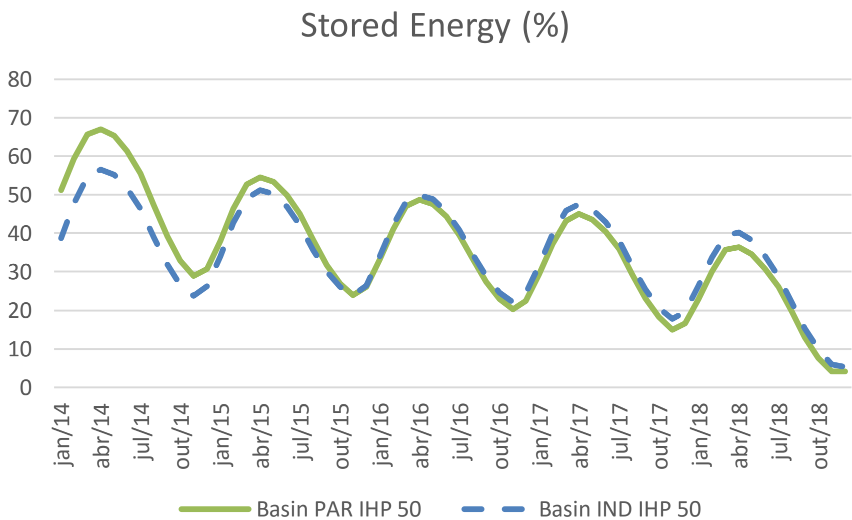
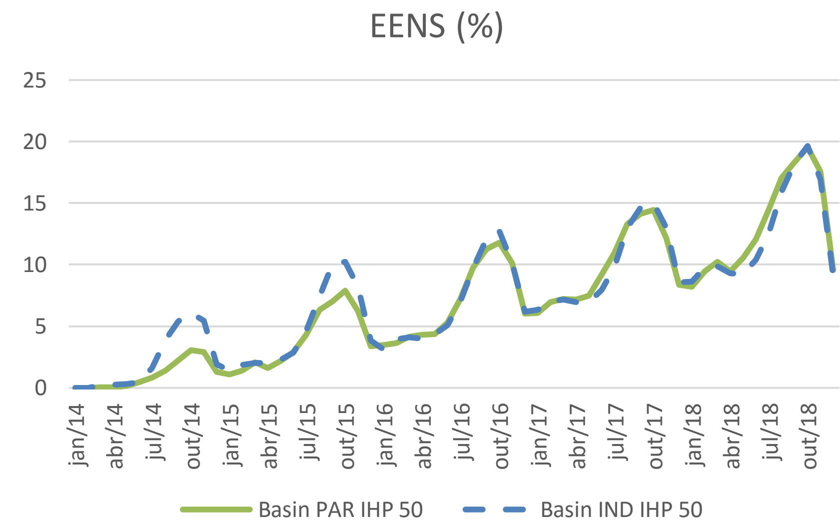


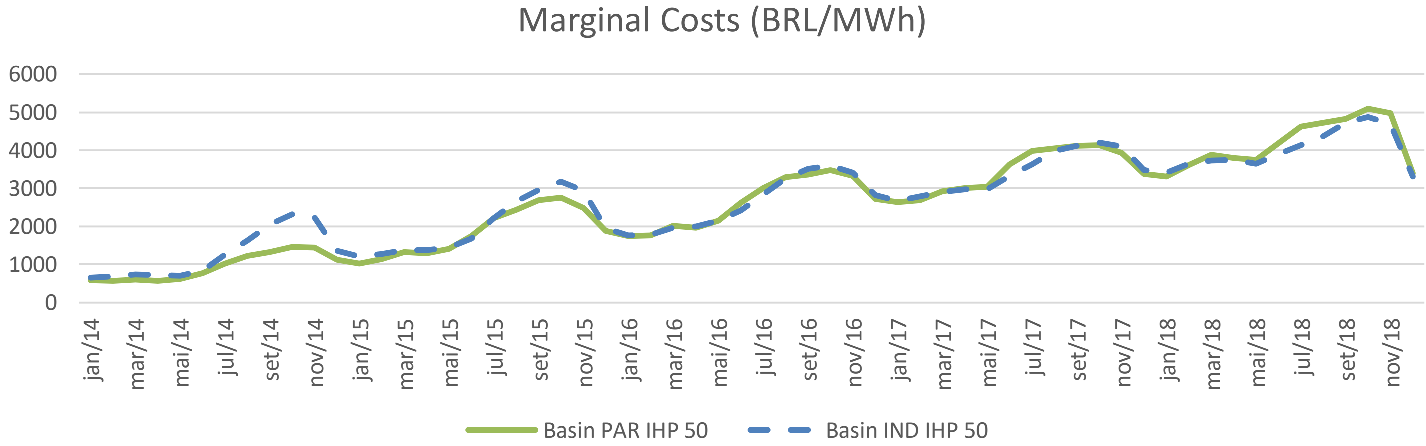


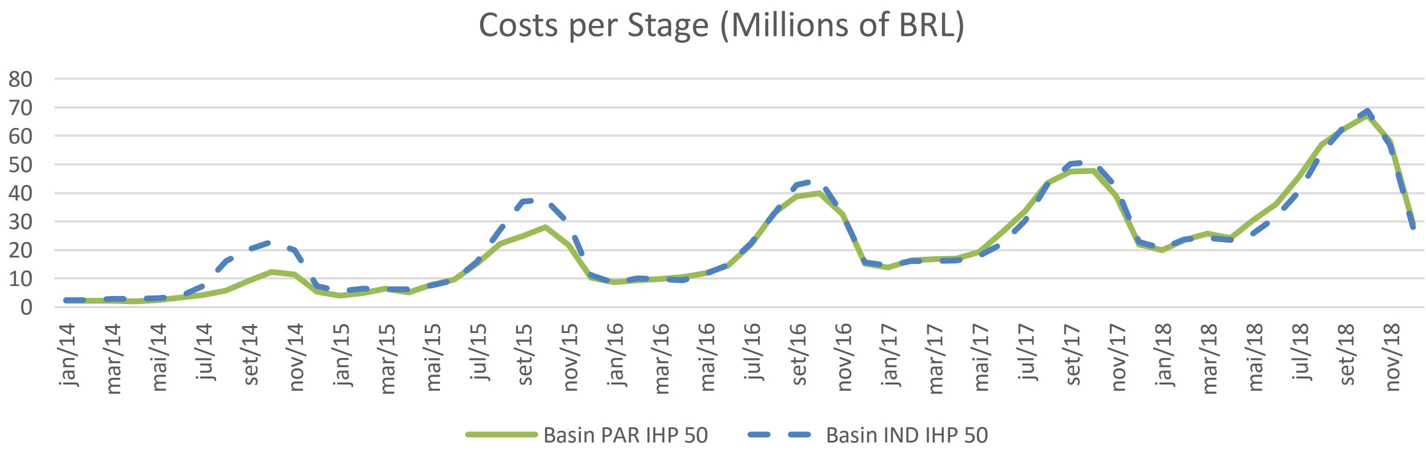
Expected Operation Cost (Billions of BRL – R$)

	1 Year	3 Years	5 Years
PAR	46.68	274.80	651.96
IND	75.67	336.32	704.86
Increase	77%	22%	8%

Comparing Independence and PAR

Simulation using 77 historical series of 5 years





Final remarks

- Results indicate that...
  - EER seems to be **optimistic**
  - Stagewise independent model may be useful
  - It is possible to get **a better policy** than what is done in Brazil
- Approximations are necessary, but...
  - They are not always the **best approach**
  - **Sub optimality** is a reality in multistage SP
- This is an ongoing project and next steps are...
  - Explore **new methods to enhance the solution** quality in limited time
  - Make a **rolling horizon comparison**

Thank you!

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