On the modeling and solution strategy of the Long Term Hydrothermal Scheduling

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Several **applications are Large Scale**

The strategy is to decompose in **smaller subproblems**

We keep a connection between the subproblems that is **improved iteratively**

There are several algorithms for **Stochastic Programming**

- We are interested in **SDDP**
Large Scale Optimization

- Scenario Tree
Large Scale Optimization

- Scenario Tree

- Sampling ➔ Monte Carlo (MC)
Large Scale Optimization

- It is not possible to solve the whole problem
  - Due to the number of stages
An alternative is to use a sampling algorithm

Stochastic Dual Dynamic Programming - SDDP

- Sample scenarios → Monte Carlo
An alternative is to use a sampling algorithm

- Stochastic Dual Dynamic Programming - SDDP
  - Sample scenarios → Monte Carlo
SDDP

- SDDP has two main steps:
  - Forward
  - Backward
In the backward we add information to our policy

- Future Cost Function (FCF)
- Benders Cuts or, simply, Cuts
SDDP - Summary

- Breaks the problem into node and scenarios

- **Sampling scenarios** allow building a policy for multi-stage scenario trees

- Connection is through a **Cost-to-go Function**
  - Built as a piecewise linear approximation over the state space

- Requires longer if
  - the number of stages grow
  - the number of random variables grow
  - the number of state variables grow
SDDP - Summary

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- **Sampling scenarios** allow building a policy for multi-stage scenario trees

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  - the number of random variables grow
  - the number of state variables grow

What if we grow the random variables and the state space?
Recent advances in Sampling Methods...

- such as,
  - Different **tree traversing** strategies
  - **Cut selection** to reduce LP size
  - **Risk aversion** methods
  - Scenario **reduction techniques** and **variance reduction sampling**
  - New strategies for **parallelization**
  - Etc...

- Led us to believe that it is possible to find **a solution/policy**
Recent advances in Sampling Methods...

...such as,

- Different **tree traversing** strategies
- **Cut selection** to reduce LP size
- **Risk aversion** methods
- Scenario **reduction techniques** and variance reduction sampling
- New strategies for **parallelization**
- Etc...

... Led us to believe that it is possible to find a solution/policy

But, how good is it? Compared to what?
Brazilian Electric System

Main characteristics (Dec. 2014)

- **194 Hydro Plants**
- **86.7 GW Installed**
- **1.774 Thermal Units**
- **39.7 GW Installed**
- **approx. 110,000 km**

- **Storage Capacity**
- **Run of River**

- **Demand of Electricity (GW)**

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Brazilian Planning Chain

Long Term:
• 10 years planning horizon
• Simplified Modeling
• Inflows are uncertain

Medium Term:
• 2 to 6 months horizon
• More detailed modeling
• Inflows are uncertain

Short Term:
• 48 hours to 1 week horizon
• Extremely detailed modeling
• Inflows are assumed known

Stochastic Simplified Modeling
Deterministic Detailed Modeling
Hydro Modeling

How to model the Hydro Plants...

How to model the Random Variables...

How to model the Inflow Process...
Hydro Modeling

How to model the Hydro Plants...

How to model the Random Variables...

How to model the Inflow Process...
Individual and EERs

Energy Equivalent Reservoirs (EERs)

- Larger State and Random Space

Optimistic Approximation!

- Energy Inflow
- Gross Run of River Energy
- Controllable Energy
- Evaporated Energy
- Spillage Energy
- Minimum Discharge Energy
- Hydro Energy
- Run of River Energy
- Loss of Run of River Energy
- Hydro Plant (Reservoir)
- Hydro Plant (Run of River)
- Electricity

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Energy Equivalent Reservoirs (EERs)

List of Attributes

- Maximum Storage Energy
- Minimum Storage Energy
- Natural Energy Inflow
- Controllable Energy Inflow and Run-of-River Energy Inflow
- Loss of Run-Of-River Energy Inflow
- Correction Factor of Controllable Energy Inflow
- Minimum Discharge Energy
- Evaporated Energy
- Maximum Energy Generation
- Small Hydroplants Energy
- Production Before Full Commitment
- Fulfilling Minimum Stored Energy
- Controllable Inflow Deviation Energy
- Run-of-River Inflow Deviation Energy
Energy Equivalent Reservoirs (EERs)

**Maximum Storage Energy**

Formulation:

\[
e^{\text{max}} = \sum_{i \in R} \left[ \left( \frac{v_{i}^{\text{max}}}{2.63} - \frac{v_{i}^{\text{min}}}{2.63} \right) \left( \sum_{j \in J} \rho_{j} h_{j}^{\text{avg}} \right) \right]
\]
When considering inflows per Plant and PAR(1)

<table>
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<tr>
<th></th>
<th>EER</th>
<th>IHP</th>
<th>Difference (%)</th>
</tr>
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<tr>
<td>Constraints*</td>
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<td>163</td>
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</table>

* No cuts taken into account.
Hydro Modeling

How to model the Hydro Plants...

How to model the Random Variables...

How to model the Inflow Process...
Random Variable modeling

- **Individual**
  - Larger number of random variables

- **Per Basin**
  - Smaller number of random variables
But, what if we consider inflows per Basin...

<table>
<thead>
<tr>
<th></th>
<th>IHP Hydro</th>
<th>IHP Basin</th>
<th>Difference (%)</th>
</tr>
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<tbody>
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<td>Constraints*</td>
<td>163</td>
<td>163</td>
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</tr>
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* No cuts taken into account.
How to model the Hydro Plants...

IHP

Individually

Stage wise Independent

PAR

Per Basin

Stage wise Independent

PAR

EER

Per EER

Stage wise Independent

PAR

How to model the Random Variables...

How to model the Inflow Process...

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Inflow scenario generation model

- Periodic Auto Regressive model – PAR(\(p\))
  - Order identification using ACF/PACF
  - Auto Regressive coefficients are computed using ACF
  - We fit a 3-Parameter LogNormal to the historical residuals

\[
\left( \frac{y_{rt} - \mu_{rm}}{\sigma_{rm}} \right) = \phi_{t1} \left( \frac{y_{r,t-1} - \mu_{r,m-1}}{\sigma_{r,m-1}} \right) + \ldots + \phi_{rp} \left( \frac{y_{r,t-p} - \mu_{r,m-p}}{\sigma_{r,m-p}} \right) + K_{rt}^{\omega_r}
\]

- Independent model
  - We fit a 3-Parameter LogNormal to the historical inflows

\[
y = \exp(\xi x + \mu_x) + \Lambda.
\]
How to model the Hydro Plants...

How to model the Random Variables...

How to model the Inflow Process...
Computational Experiment

- **Base case**
  - IHP - Random variables *per Basin* and *PAR(p)*

- **Comparison**
  - EER - Random variables *per EER* and *PAR(p)*
  - IHP - Random variables *per Basin* and *Stagewise Independent*

- **Configuration**
  - SDDP algorithm
  - 30% initial condition (volumes)
  - 5-year horizon with monthly stages
  - 50 realizations per stage
  - 30 iterations with 200 scenarios per iteration
  - 10 threads (20 scenarios per thread in each iteration)
Cost-to-go Function is computed for the following conditions:

- IHP - Random variables per Basin and PAR(p)
- EER - Random variables per EER and PAR(p)
- IHP - Random variables per Basin and Stagewise Independent

But all simulations are based on

- IHP - Random variables per Hydro Plant and PAR(p)

2000 Outsample Scenarios

77 Historical Series
Lower and Upper Bounds
Lower and Upper Bounds

9 iterations
1,800 cuts

15x
Comparing IHP and EER

Simulation using 2000 outsample considering Individual Inflows

Stored Energy (%)

EENS (%)

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Comparing IHP and EER

Simulation using 2000 outsampel considering Individual Inflows

Expected Operation Cost (Billions of BRL – R$)

- EER 30.03
- IHP 34.79
- IHP 1800 35.04

- EER 130.40
- IHP 60.59
- IHP 1800 61.31

14% 1% 54% 1%
Comparing IHP and EER

Simulation using 2000 outsample considering Individual Inflows

Marginal Cost (BRL/MWh)

Cost per Stage (Millions of BRL)
Lower and Upper Bounds
Lower and Upper Bounds

![Graph showing cost over time for different basins. The x-axis represents time in hours, ranging from 0.03 to 32.00. The y-axis represents cost in Billions of BRL, ranging from 0 to 250. The graph includes lines for Basin IND IHP 50 UB, Basin IND IHP 50 LB, Basin PAR IHP 50 UB, and Basin PAR IHP 50 LB. There is a red box highlighting the data point at 8x.]
Comparing Independence and PAR

Simulation using 77 historical series of 5 years

Stored Energy (%)

EENS (%)

Basin PAR IHP 50

Basin IND IHP 50

Basin PAR IHP 50

Basin IND IHP 50
Comparing Independence and PAR

Simulation using 77 historical series of 5 years

Stored Energy (%)

EENS (%)

Expected Operation Cost (Billions of BRL – R$)

1 Year

PAR 46.68

IND 75.67

3 Years

PAR 274.80

IND 336.32

5 Years

PAR 651.96

IND 704.86

77%

22%

8%
Simulation using 77 historical series of 5 years
Final remarks

- Results indicate that...
  - EER seems to be **optimistic**
  - Stagewise independent model may be useful
  - It is possible to get a **better policy** than what is done in Brazil

- Approximations are necessary, but...
  - They are not always the **best approach**
  - **Sub optimality** is a reality in multistage SP

- This is an ongoing project and next steps are...
  - Explore **new methods to enhance the solution** quality in limited time
  - Make a **rolling horizon comparison**
Thank you!

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