Multistage Stochastic Programming: A Modeling and Algorithmic Perspective

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Outline

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Two stage stochastic programming

- Started in the 50's. Important early works: Dantzig ('55), Beale ('55), Walkup and Wets ('67).
- Increased interest in the last 20 years due to computational advances.
- Mature field: Kall and Wallace '94, Birge and Louveaux '97, Shapiro et al. '09.
- Applications: Finance, Energy, Transportation, Production Planning, Telecommunications, Forestry, ...

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Two stage stochastic programming

decision x $\rightsquigarrow$ realization ξ $\rightsquigarrow$ Recourse action y.

$$\min_{x \in X} \{cx + \mathbb{E}[Q(x,\xi)]\},$$

where

$$Q(x,\xi) = \min_{y \in Y} \{qy | Tx + Wy \geq h\}.$$

Risk aversion

- Minimize the expected cost is just one possible criterion.
- What if bad outcomes are extremely undesirable?
- In Finance, one would like to be protected against extreme losses
- In Energy, one would like to have a policy against severe droughts, or blackouts

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Conditional Value-at-Risk

- The Value-at-Risk:

$$\mathrm{VaR}_\alpha[X] = \inf\{x : \mathbb{P}(X \leq x) \geq 1 - \alpha\}, \quad \alpha \in (0, 1).$$

- Formally, we define

$$\mathrm{CVaR}_\alpha[X] = \inf_{t \in \mathbb{R}} \left\{ t + \frac{1}{1-\alpha} \mathbb{E}\left[X - t\right]_+ \right\} =$$
$$(\text{ Cont. case }) = \mathbb{E}\left[X \mid X > \mathrm{VaR}_\alpha\right].$$

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Coherent risk measures

1) $\rho(X + c) = \rho(X) + c$.

2) $X \leq Y \Rightarrow \rho(X) \leq \rho(Y)$.

3) $\rho(\lambda X) = \lambda\rho(X)$ for $\lambda \geq 0$.

4) $\rho(X + Y) \leq \rho(X) + \rho(Y)$.

A risk measure that satisfies axioms 1) – 4) is called *coherent*.

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Risk aversion in two stage SP

- In the two stage context risk aversion is well understood
- Stochastic Dominance: Dentcheva & Ruszczyński '03 '04, Hu & Homem-de-Mello, Roman '06, ...
- Risk measures: Rockafellar & Uryasev '00, Schulz & Tiedemann '03 '06, Ahmed '06, Miller and Ruszczyński '11, Noyan '12, ...
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- Stochastic Dynamic Programming. Or ...
- Markov Decision Process, or
- Multistage Stochastic Programming, or
- Intertemporal Consumption, or
- Life-Cycle Consumption, or...

They all want to solve the same problem: optimal decision making over time under uncertainty.

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Examples

Many problems can be framed as multistage problems:

- Hydroelectric energy planning: How much energy to produce/store in each month, given that water inflows are uncertain?
- Portfolio selection: How much money should I put on each investment every month, knowing that future returns are uncertain?
- Revenue management: Which products (e.g., fare classes) should be made available at each time period, given that future demand is uncertain?

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Algorithms

- Popular algorithms include the Nested L-Shaped (Birge ’85), SDDP (Pereira and Pinto ’91), Progressive Hedging (Rockafellar and Wets ’91), SAA (Shapiro ’03, ’06), ADP (Powell ’07).
- The effectiveness of each algorithm is highly problem dependent.
- General purpose algorithms are not readily applicable.

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General formulation of MSSP with random RHS

Assume $\{\xi_1, \ldots, \xi_T\}$ is a stochastic process, ξ_0 is a constant.

$$\max \ \mathbb{E}_{\xi_1,\ldots,\xi_T}\left[c_0' x_0 + c_1' x_1 + \ldots + c_T' x_T\right]$$

subject to [MSSP]

$$\begin{aligned}
A_0 x_0 &\leq \xi_0 \\
A_1 x_1 &\leq \xi_1 - B_0 x_0 \\
&\vdots \\
A_T x_T &\leq \xi_T - \sum_{m=0}^{T-1} B_m x_m,
\end{aligned}$$

x_t depends only on $\xi_0, \ldots, \xi_t$.

Recursive formulation of MSSP

$$\max \; c_0^T x_0 + \mathbb{E}_{\xi_1}\left[Q_1(x_0, \xi_1)\right]$$

subject to [MSSP-R]

$$A_0 x_0 \leq \xi_0.$$

The function Q_1 is defined recursively as

$$Q_t(x_0, \ldots, x_{t-1}, \xi_1, \ldots, \xi_t) = \max_{x_t} \; c_t^T x_t + \mathbb{E}_{\xi_{t+1}}\left[Q_{t+1}(x_0, \ldots, x_t, \xi_1, \ldots, \xi_{t+1}) \,|\, \xi_1, \ldots, \xi_t\right]$$

subject to

$$A_t x_t \leq \xi_t - \sum_{m=0}^{t-1} B_m x_m,$$

$t = 1, \ldots, T$. Also, $Q_{T+1} \equiv 0$.

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Risk averse multistage stochastic programming

- There is no obvious way of formulating the problem
- Some issues that are absent (or not so relevant) in two-stage case complicate matters in the multistage setting
- Our goal is to discuss one of those issues (consistency), and to understand some possible frameworks
- We focus on a specific formulation, and apply our findings to a pension fund problem

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General framework

In our context, multi-period risk measures are those applied to real-valued functions of the stochastic process $\{\xi_t\}_{t=1}^T$. To simplify notation, let

$$Z_t := f_t(x_t, \xi_t), Z := (Z_1, \ldots, Z_T).$$

The multi-period risk measure is denoted by $\mathbb{F}(Z)$.

$$\min_{x_1,\ldots,x_T} \mathbb{F}\left(f_1(x_1,\xi_1),\ldots,f_T(x_T,\xi_T)\right)$$
$$\text{s.t.} \quad x_t \in \mathcal{X}_t\left(x_{[t-1]},\xi_{[t]}\right), \quad t = 1,\ldots,T.$$

Conditional risk function:

$$\mathbb{F}^{Z_1,\ldots,Z_t}(Z)(\omega) := \tilde{\mathbb{F}}(G_{Z\,|\,Z_1=z_1,\ldots,Z_t=z_t}).$$

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Examples

Expected value:

$$\mathbb{F}(Z_1, \ldots, Z_T) = \mathbb{E}[Z_1 + \ldots + Z_T]$$

Worst-case risk measure:

$$\mathbb{F}(Z_1, \ldots, Z_T) = \text{ess sup}(Z_1 + \ldots + Z_T)$$

Separated risk per stage:

$$\mathbb{F}(Z) = Z_1 + \rho_2(Z_2) + \ldots + \rho_T(Z_T)$$

Global risk:

$$\mathbb{F}(Z) = \rho(Z_1 + Z_2 + \ldots + Z_T)$$

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Consistency

There are several definitions in the literature (Ruszczyński '10, Kovacevic and Pflug '14, Detlefsen and Scandolo '05, Cheridito et al. '06, Bion-Nadal '08, Shapiro '09, Carpentier et al. '12, Rudloff et al. '14, Xin et al. '13, Pflug and Pichler '14, Eckstein '16).

Informal definition

When you solve the problem at time 1 and again at time t, some optimal solution obtained at time 1–calculated at the scenario that actually occurred between 1 and t–will also be optimal at time t.

Even more informal definition

The decision you make today should agree with some optimal plan made yesterday, given what was observed today.

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Formal definition

$$\min_{x_t,\ldots,x_T} \mathbb{F}^{\hat{\xi}_{[t]}} \left(f_t(x_t,\xi_t),\ldots,f_T(x_T,\xi_T)\right) \quad (1)$$

$$\text{s.t.} \quad x_\tau \in \mathcal{X}_\tau\left(\hat{x}_{[t-1]}, x_t,\ldots,x_{\tau-1},\hat{\xi}_{[t]},\xi_{t+1},\ldots,\xi_\tau\right), \quad \tau = t,\ldots,T.$$

IOP

We say that the *inherited optimality property (henceforth called IOP)* holds if for any t such that $1 < t \le T$ and any realization $\hat{\xi}_1,\ldots,\hat{\xi}_t$, there exists an optimal solution x^* of the multistage problem such that the solution "inherited" from x^* at $\hat{\xi}_2,\ldots,\hat{\xi}_t$ coincides with an optimal solution of (1) for those t, $\hat{\xi}$, and $\hat{x} := x^*$.

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We say that the multi-period risk measure $\mathbb{F}$ is *consistent* if the IOP holds for any particular instance of that problem.

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A 3-stage inventory problem

- Assume you are a retailer who sells one product and needs to decide now how much inventory to buy, at price $c = \$2$.
- There will be two selling opportunities: in the second stage the product can be sold at price $s_1 = \$3$ and on the third stage the product can be sold for $s_2 = \$10$.
- At the end of the horizon unsold units are discarded.
- Demand is given by a binary tree.

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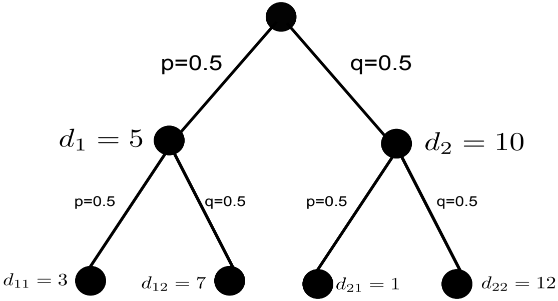
A scenario tree

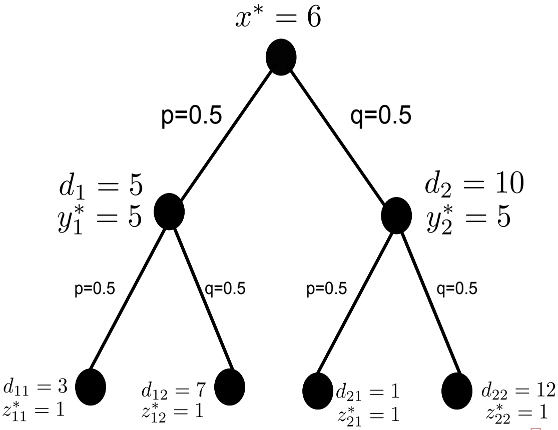


Separated risk per stage

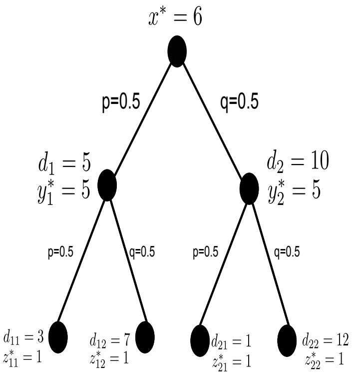
Using $\mathcal{R}(Z) = Z_1 + \rho_2(Z_2) + \ldots + \rho_T(Z_T)$, we have

$$
\begin{aligned}
\min\; & cx + \rho_1\left(-s_1 y\right) + \rho_2\left(-s_2 z\right) \\
\text{s.t. } & y \le D, \\
& y \le x, \\
& z + y \le x, \\
& y \le D.
\end{aligned}
$$

Solution


$x^* = 6$
p=0.5
q=0.5
$d_1 = 5$
$y_1^* = 5$
$d_2 = 10$
$y_2^* = 5$
p=0.5
q=0.5
p=0.5
q=0.5
$d_{11} = 3$
$z_{11}^* = 1$
$d_{12} = 7$
$z_{12}^* = 1$
$d_{21} = 1$
$z_{21}^* = 1$
$d_{22} = 12$
$z_{22}^* = 1$

Solution



$x^* = 6$
p=0.5
q=0.5
$d_1 = 5$
$y_1^* = 5$
$d_2 = 10$
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p=0.5
q=0.5
p=0.5
q=0.5
$d_{11} = 3$
$z_{11}^* = 1$
$d_{12} = 7$
$z_{12}^* = 1$
$d_{21} = 1$
$z_{21}^* = 1$
$d_{22} = 12$
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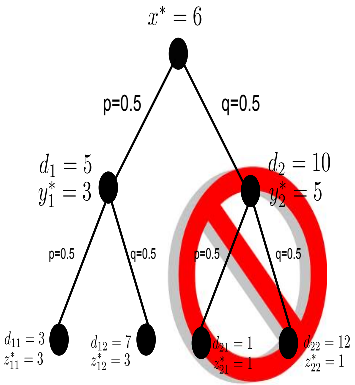

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- In Homem-de-Mello and P. ’15 it is shown that the global risk measure is also inconsistent.
- Are there examples of nontrivial consistent risk measures?
- If so, can the resulting formulations actually be solved?
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Consistent risk measures

Theorem

Consider the risk measure given by

$$\mathcal{R}(Z_1 + Z_2 + \ldots + Z_T) = \rho_2 \circ \rho_3^{\xi_{[2]}} \circ \cdots \circ \rho_T^{\xi_{[T-1]}}(Z_1 + \ldots + Z_T).$$

If each $\rho_t^{\xi_{[t-1]}}$ is translation-invariant and monotone, then $\mathcal{R}$ is consistent.

Corollary

Nested risk measures, the risk neutral risk measure $\mathcal{R}(Z_1, \ldots, Z_T) = \mathbb{E}[Z_1 + \ldots + Z_T]$, and the worst-case risk measure $\mathcal{R}(Z_1, \ldots, Z_T) = \mathit{esssup}(Z_1 + \ldots + Z_T)$ are also consistent.

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Nested risk measures

$$\max \; c_0^T x_0 + \rho_{\xi_1}\left[Q_1(x_0,\xi_1)\right]$$

subject to [Risk-MSSP]

$$A_0 x_0 \le \xi_0.$$

The function Q_1 is defined recursively as

$$Q_t(x_0,\ldots,x_{t-1},\xi_1,\ldots,\xi_t) =$$

$$\max_{x_t} \; c_t^T x_t + \rho_{\xi_{t+1}}\left[Q_{t+1}(x_0,\ldots,x_t,\xi_1,\ldots,\xi_{t+1}) \,|\, \xi_1,\ldots,\xi_t\right]$$

subject to

$$A_t x_t \le \xi_t - \sum_{m=0}^{t-1} B_m x_m,$$

$t = 1,\ldots,T$. Also, $Q_{T+1} \equiv 0$.

Nested CVaR

A series of recent publications consider the case of nested risk measures in which each $\rho^{\xi_{[t-1]}}$ is defined as the Conditional CVaR:

$$\rho^{\xi_{[t-1]}}(Z_t) := \mathrm{CVaR}_{\alpha_t}^{\xi_{[t-1]}(Z_t)} = \min_{\eta_t \in \mathbb{R}} \left\{ \eta_t + \frac{1}{1-\alpha_t} \mathbb{E}[(Z_t - \eta_t)_+ | \xi_{[t-1]}] \right\}.$$

Advantages:

- It is consistent, and Bellman-type algorithms for DP can be employed.
- Linearizing the CVaR leads to a standard MSSP, which can be solved by existing methods developed for risk neutral problems, e.g., SDDP: Guigues and Römisch '12, Guigues and Sagastizábal '12, Philpott, De Matos, Finardi '13, Shapiro et al. '13, Kozmík and Morton '14, Piazza and P. '15.

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- We consider a class of risk measures called expected conditional risk measures (ECRMs), which first appeared in Pflug and Ruszczynski '05

$$ECRM(Z) = Z_1 + \rho_2(Z_2) + \mathbb{E}_{\xi_{[2]}}\left[\rho_3^{\xi_{[2]}}(Z_3)\right] + \ldots + \mathbb{E}_{\xi_{[T-1]}}\left[\rho_T^{\xi_{[T-1]}}(Z_T)\right]$$

- It is a promising candidate for several reasons:
  1. (Homem-de-Mello and P., '15) If the risk measure ρ is translation invariant and monotone, then the corresponding ECRM is consistent.
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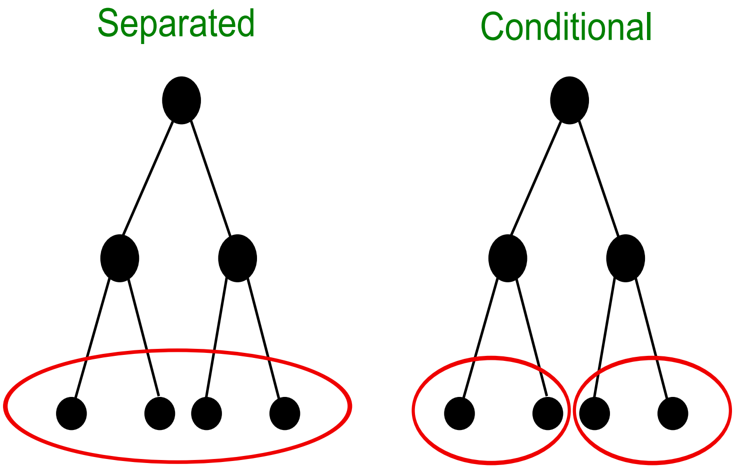
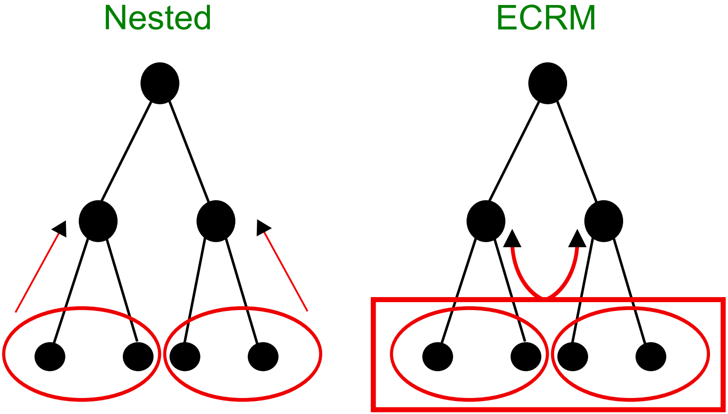
Separated versus conditional



Nested versus ECRM



$$\rho_2(Z_2 + \rho_3^{\xi_{[2]}}(Z_3))$$

$$E_{\xi_{[2]}}[\rho^{\xi_{[2]}}(Z_3)]$$

Outline

A numerical example

We consider the Dutch pension fund problem described in Klein Haneveld, Streutker and van der Vlerk (2010).

- The fund sponsor has to maintain the ratio between assets and liabilities above some pre-specified threshold at every time period.
- To achieve this goal, three sources of income can be used:
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- We consider a problem with 4 stages, a scenario tree with 1000 scenarios, with 10 bifurcations per node (thanks to M. van der Vlerk for the data)
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α	stocks	bonds	real estate	cash	c_1	Z_1
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- The $(.95, 0, 0)$ solution invests only 30% in stocks, as opposed to an average of 48% of other solutions.
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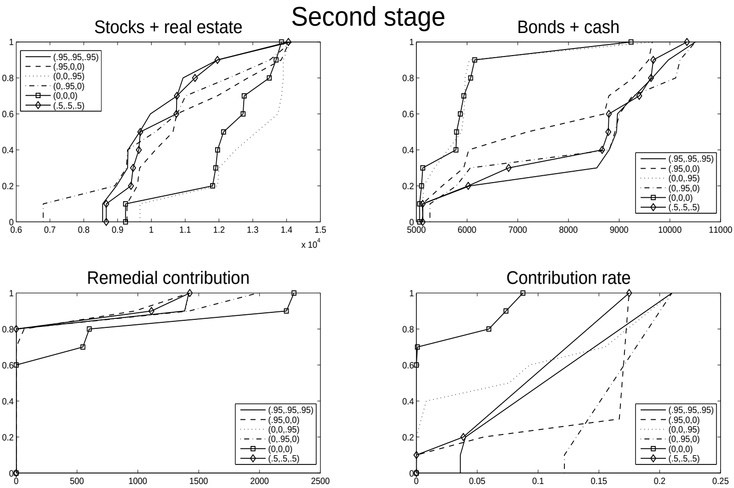
A representation of the second stage solution



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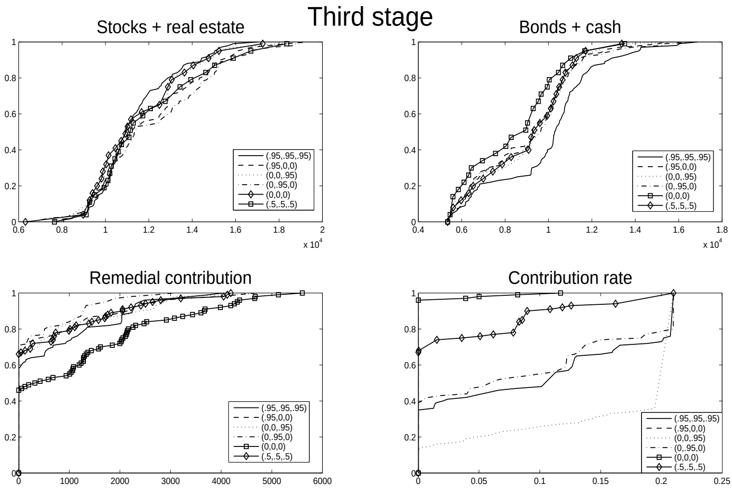

Dominance

- The remedial contribution for the risk neutral solution (RNS) first-order stochastically dominates (FOSD) all other solutions in both stages!
- In other words, for every value of x the probability of having losses greater than or equal to x will be larger for the RNS.
- The (.95, .95, .95) solution FOSD (0,.95,0) and SOSD (.95,0,0).
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- Bonds and cash: the RNS is second-order stochastically dominated by all other solutions in both stages while the (.95,.95,.95) SOSD all other solutions in the third stage.
- The probability of investing less than $\$10^4$ on risky assets is on average 45% for risk averse solution, while it is only 10% for the RNS.
- The (.5, .5, .5) is a interesting case: its curves for remedial contribution and contribution rate lie roughly in between all curves. As expected?

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Outline

Conclusions

- While the is some consensus on how two stage risk averse stochastic programming should be modelled, there is no standard way of modeling multistage risk averse problems.
- Much of the literature focuses on nested CVaR formulations. Great research opportunity!
- We have discussed an alternative approach, which we call ECRM, that addresses some issues that appear with nested formulations.
- When the CVaR is chosen, the problem can represented as a risk-neutral problem on a “lifted” space, so standard algorithms can be used.
- Moreover, since risk measures are applied in a stage-wise fashion, it is easier to control how risk is being measured via the values of α_t.
- Key questions, from a practical viewpoint: a)What is the appropriate risk measure for my problem? b) How can I generate scenarios adapted to some risk measure?

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Contact information

Thank you!

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Reference: Homem-de-Mello, T. and P. Risk aversion in multistage stochastic programming: a modeling and algorithmic perspective. EJOR '16.

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