Optimality of Doubly Reflected Lévy Processes in Singular Control

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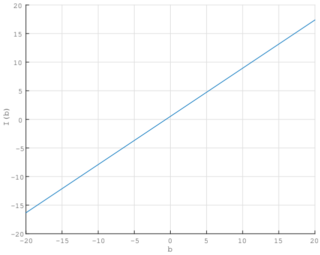
Explicit Solutions in Stochastic Control

- One Parameter Case
  - Optimal stopping – Mordecki (FS, 2002), Asmussen, Avram & Pistorius (2004, SPA), Hilberink & Rogers (FS, 2002)
  - Singular control – w/o fixed costs – Avram, Palmowski, Pistorius (AAP, 2007), Loeffen (AAP, 2008), Bayraktar, Kyprianou & Y. (ASTIN, 2014).
- Two Parameter Case
  - Impulse control – w/ fixed costs – Bensoussan, Liu & Sethi (SICON, 2005), Loeffen (IME, 2011), Bayraktar, Kyprianou & Y. (IME, 2013), Y. (MOR, forthcoming).
  - Games b/w two players – Egami, Leung & Y. (SPA, 2013), Hernandez-Hernandez & Y. (SPA, 2015)
  - Two-sided singular control – Baurdoux & Y. (SPA, 2015, This talk)

One Parameter Case

Suppose the NPV $v_a(x)$ can be computed.

- Smooth/Continuous fit (or first-order) condition to derive the candidate threshold a^*.
- Typically one obtains a monotone function $a \mapsto \Gamma(a)$:

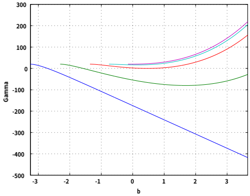


- Obtain the root a^* s.t. $\Gamma(a^*) = 0$.
- Then verify the optimality of v_{a^*}.

Two Parameter Case

Suppose the NPV $v_{a,b}(x)$ can be computed.

- Two Smooth/Continuous fit (or two first-order) conditions to derive the candidate thresholds a^* and b^*.
- Typically one obtains a function like $(a, b) \mapsto \Gamma(a, b)$ and the conditions are equivalently to $\Gamma(a^*, b^*) = \partial\Gamma(a^*, b^*)/\partial b = 0$.



- Consider $b \mapsto \Gamma(a, b)$ and obtain the curve s.t. it starts at a^* and touches and is tangent to the x-axis at b^*.
- Then verify the optimality of v_{a^*,b^*}.

Objective

- As an example where two parameters characterize the optimal strategy, we consider the two-sided singular control problem.
- BM model has been studied by e.g. Constantinides & Richard (OR, 1978), Sulem (Math OR, 1986) for quadratic/ linear f.
- We want to minimize

$$v^{\pi}(x) := \mathbb{E}_x \Big[\int_0^{\infty} e^{-qt} f(Y_t^{\pi}) \mathrm{d}t + \int_{[0,\infty)} e^{-qt} \left(C_U \mathrm{d}U_t^{\pi} + C_D \mathrm{d}D_t^{\pi} \right) \Big],$$

with the controlled process

$$Y_t^{\pi} := X_t + U_t^{\pi} - D_t^{\pi},$$

for the case X is a spectrally negative Lévy process.
- Show the optimality of doubly reflected Lévy process of Pistorius (SPA, 2003).

Outline

1. Model
2. Review of (doubly reflected) spectrally negative Lévy processes and scale functions.
3. Candidate threshold levels – and their existence.
4. Verification of optimality.
5. Numerical results

If time allowed, I will talk about the optimality of (s, S)-policy in the impulse control with fixed costs.

Model

- $(\Omega, \mathcal{F}, \mathbb{P})$ hosting a *spectrally negative Lévy process* X.
- An admissible strategy $\pi := \{(U_t^\pi, D_t^\pi); t \geq 0\}$ – nondecreasing, right-continuous and adapted processes such that $U_{0-}^\pi = D_{0-}^\pi = 0$ and $\mathbb{E}_x\Big[\int_{[0,\infty)} e^{-qt}(\mathrm{d}U_t^\pi + \mathrm{d}D_t^\pi)\Big] < \infty$.
- We want to minimize

$$v^\pi(x) := \mathbb{E}_x\Big[\int_0^\infty e^{-qt} f(Y_t^\pi)\mathrm{d}t + \int_{[0,\infty)} e^{-qt}\left(C_U \mathrm{d}U_t^\pi + C_D \mathrm{d}D_t^\pi\right)\Big],$$

with the controlled process

$$Y_t^\pi := X_t + U_t^\pi - D_t^\pi,$$

for the case X is a spectrally negative Lévy process.

Assumptions

1. The unit proportional costs C_U and C_D can be negative but must satisfy

$$C_U + C_D > 0.$$

2. We assume that $\mathbb{E}[X_1] = \psi'(0+) \in (-\infty, \infty)$.
3. About f (similar assumptions as in Bensoussan et al. (2005))
   - convex (can be generalized slightly);
   - grows (or decreases) at most polynomially;
   - There exists a number $\bar{a} \in \mathbb{R}$ such that the function

$$\tilde{f}(x) := f(x) + C_U q x, \quad x \in \mathbb{R},$$

   is increasing on $(\bar{a}, \infty)$ and is decreasing on $(-\infty, \bar{a})$.

Spectrally Negative Lévy Processes

- Let X be a spectrally negative Lévy process with a Laplace exponent:

$$\begin{aligned}\psi(s) &:= \log \mathbb{E}\left[e^{sX_1}\right] \\ &= cs + \frac{1}{2}\sigma^2 s^2 + \int_{(-\infty,0)} (e^{sz} - 1 - sz1_{\{-1<z<0\}})\nu(\mathrm{d}z),\end{aligned}$$

  such that $\int_{(-\infty,0)} (1 \wedge z^2)\nu(\mathrm{d}z) < \infty$.
- It has paths of bounded variation if and only if $\sigma = 0$ and $\int_{(-1,0)} z\,\nu(\mathrm{d}z) < \infty$.
- We exclude the case X is a subordinator.

Scale Functions

- Recall that X is a spectrally negative Lévy process with Laplace exponent $\psi(s) = \log \mathbb{E}\left[e^{sX_1}\right]$.
- Fix any $q > 0$, there exists a function called the q-scale function

$$W^{(q)} : \mathbb{R} \to [0, \infty),$$

which is zero on $(-\infty, 0)$, continuous and strictly increasing on $[0, \infty)$, and is characterized by the Laplace transform:

$$\int_0^\infty e^{-sx} W^{(q)}(x) \mathrm{d}x = \frac{1}{\psi(s) - q}, \qquad s > \Phi(q),$$

where

$$\Phi(q) := \sup\{\lambda \geq 0 : \psi(\lambda) = q\}.$$

Scale Functions (Cont'd)

Let us define the first down- and up-crossing times, respectively, by

$$\tau_a^- := \inf\{t \geq 0 : X_t < a\}$$
$$\tau_b^+ := \inf\{t \geq 0 : X_t > b\}.$$

Then we have for any $b > 0$

$$\mathbb{E}_x\left[e^{-q\tau_b^+} 1_{\{\tau_b^+ < \tau_0^-\}}\right] = \frac{W^{(q)}(x)}{W^{(q)}(b)},$$
$$\mathbb{E}_x\left[e^{-q\tau_0^-} 1_{\{\tau_b^+ > \tau_0^-\}}\right] = Z^{(q)}(x) - Z^{(q)}(b)\frac{W^{(q)}(x)}{W^{(q)}(b)},$$

where

$$\overline{W}^{(q)}(x) := \int_0^x W^{(q)}(y)\mathrm{d}y,$$
$$Z^{(q)}(x) := 1 + q\overline{W}^{(q)}(x).$$

Double reflection strategies

- A doubly reflected Lévy process given by

$$Y_t^{a,b} := X_t + U_t^{a,b} - D_t^{a,b}, \quad t \geq 0,\ a < b,$$

which is reflected at two barriers a and b so as to stay on the interval $[a, b]$.

- The corresponding NPV of costs becomes

$$v_{a,b}(x) = \frac{\Gamma(a, b)}{qW^{(q)}(b-a)} Z^{(q)}(x-a) - C_U R^{(q)}(x-a) + \frac{f(a)}{q} - \int_a^x \overline{W}^{(q)}(x-y) f'(y) \mathrm{d}y.$$

For $x \geq b$, we have $v_{a,b}(x) = v_{a,b}(b) + C_D(x-b)$.

- Here

$$R^{(q)}(y) := \overline{Z}^{(q)}(y) + \frac{\psi'(0+)}{q}, \quad y \in \mathbb{R}.$$

Selection of a and b

- Define

$$\Gamma(a,b) := C_D + C_U Z^{(q)}(b-a) + f(b)W^{(q)}(0) + \int_a^b f(y) W^{(q)\prime}(b-y)\mathrm{d}y - W^{(q)}(b-a)f(a),$$
$$\gamma(a,b) := \frac{\partial}{\partial b}\Gamma(a,b-).$$

- We see that the values of (a,b) such that $\Gamma(a,b)$ and $\gamma(a,b)$ vanish simultaneously attain smoothness.

Selection of a and b (Cont'd)

- Taking a derivative and then limits

$$v'_{a,b}(b-) = C_D \quad \text{and} \quad v'_{a,b}(a+) = \frac{\Gamma(a,b)}{W^{(q)}(b-a)} W^{(q)}(0) - C_U.$$

- Taking another derivative and then limits

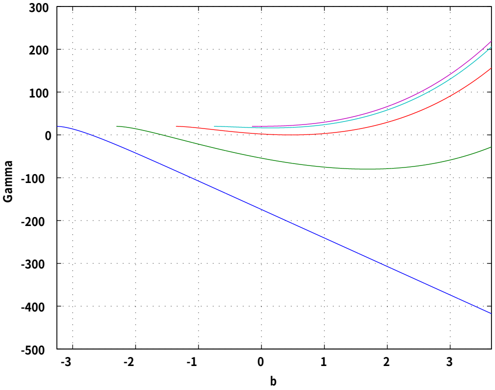
$$v''_{a,b}(b-) = \frac{\Gamma(a,b)}{W^{(q)}(b-a)} W^{(q)\prime}((b-a)-) - \gamma(a,b),$$

$$v''_{a,b}(a+) = \frac{\Gamma(a,b)}{W^{(q)}(b-a)} W^{(q)\prime}(0+) - \tilde{f}'(a+) W^{(q)}(0).$$

- Recall also that $W^{(q)}(0) = 0$ iff X is of unbounded variation.
- If $\frac{\Gamma(a,b)}{W^{(q)}(b-a)} W^{(q)\prime}((b-a)-) = \gamma(a,b) = 0$, then $v_{a,b}$ is
  - differentiable (resp. twice-differentiable) at a when X is of bounded (resp. unbounded) variation.
  - it is twice-differentiable at b.

Existence of a^* and b^*

Plots of $b \mapsto \Gamma(a, b)$ for five fixed values of a – the red one is what we want.



Candidate Thresholds

There exist a^* and b^* such that $\Gamma(a^*, x) \geq 0$ for all $x \in [a^*, \infty)$ and either **Case 1** or **Case 2** defined below holds.

Case 1 $d^* < u^*$ and

$$\Gamma(a^*, b^*) = 0.$$

Moreover, if $\gamma(a^*, b^*)$ is continuous at u^* then we also have that $\gamma(a^*, b^*) = 0$.

Case 2 $a^* \in \mathbb{R}$ and $b^* = \infty$ and

$$\lim_{b \to \infty} \frac{\Gamma(a^*, b)}{W^{(q)}(b - a^*)} = 0.$$

Verification of optimality

With our choice of (a^*, b^*),

$$v_{a^*,b^*}(x) = -c_U\Big(\frac{\psi'(0+)}{q} + x\Big) + \frac{\tilde{f}(a^*)}{q} Z^{(q)}(x - a^*) - \int_a^x W^{(q)}(x-y)\tilde{f}'(y)\mathrm{d}y.$$

Verification of optimality requires

1. $-C_U \leq v'_{a^*,b^*}(x) \leq C_D$ for all $x \in \mathbb{R}$.
2. $(\mathcal{L} - q)v_{a^*,b^*}(x) + f(x) \geq 0$ for all $x > b^*$.
3. $(\mathcal{L} - q)v_{a^*,b^*}(x) + f(x) = 0$ for $a^* < x < b^*$.
4. $(\mathcal{L} - q)v_{a^*,b^*}(x) + f(x) \geq 0$ for $x \leq a^*$.

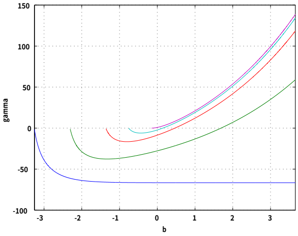
Verification of optimality (Cont'd)

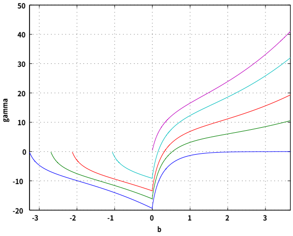
- Hard ones to prove are:
  1. $-C_U \leq v'_{a^*,b^*}(x)$ for all $x \in (a^*, b^*)$ – simple under the convexity of f,
  2. $(\mathcal{L} - q)v_{a^*,b^*}(x) + f(x) \geq 0$ for all $x > b^*$ – six page proof;

  others hold even w/o the convexity of f.
- We use the convexity of f and

$$v'_{a^*,b^*}(x) = -\Gamma(a^*, x) + C_D, \quad a^* \leq x \leq b^*.$$

- Plots of $\Gamma(a,x)$ and $\gamma(a,x) = \partial\Gamma(a,x)/\partial x$:





Meromorphic Lévy Processes

- A class of the meromorphic Lévy process Kuznetsov, Kyprianou & Pardo (AAP, 2012) admits the Lévy measure in the form:

$$\nu(\mathrm{d}z) = \sum_{j=1}^{\infty} p_j \eta_j e^{-\eta_j z} 1_{\{z>0\}} \mathrm{d}z, \quad z \in \mathbb{R},$$

 for some $\{p_k, \eta_k; k \geq 1\}$. The equation $\psi(\cdot) = q$ has a countable negative real-valued roots $\{-\xi_{k,q}; k \geq 1\}$ that satisfy the interlacing condition:
$\cdots < -\eta_k < -\xi_{k,q} < \cdots < -\eta_2 < -\xi_{2,q} < -\eta_1 < -\xi_{1,q} < 0$.

- The scale function can be written as

$$W^{(q)}(x) = \frac{e^{\Phi(q)x}}{\psi'(\Phi(q))} - \sum_{i=1}^{\infty} \frac{1}{|\psi'(-\xi_{i,q})|} e^{-\xi_{i,q}x}, \quad x \geq 0.$$

Numerical results

- The β-family introduced by Kuznetsov (AAP, 2010):

$$\psi(z) = \hat{\delta} z + \frac{1}{2}\sigma^2 z^2 + \frac{\varpi}{\beta}\left\{ B(\alpha + \frac{z}{\beta}, 1-\lambda) - B(\alpha, 1-\lambda) \right\}$$

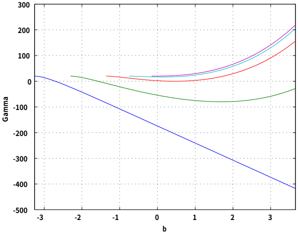
  for some $\hat{\delta} \in \mathbb{R}$, $\alpha > 0$, $\beta > 0$, $\varpi \geq 0$, $\lambda \in (0,3)\backslash\{1,2\}$ and the beta function $B(x,y)$.

- ψ is rational and hence can be inverted to obtain an analytical form of the scale function.

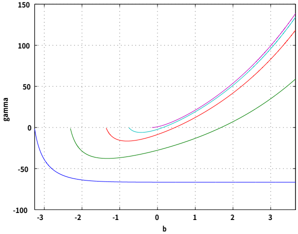
Quadratic Case

Suppose the running cost function is $f \equiv f_Q$ where

$$f_Q(x) := x^2, \quad x \in \mathbb{R}.$$

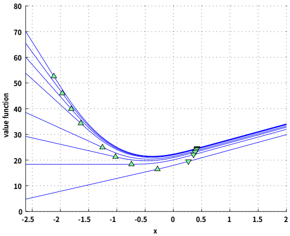


$\Gamma(a, \cdot)$

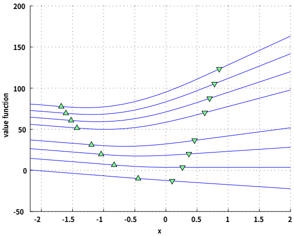


$\gamma(a, \cdot)$

Quadratic Case (Cont'd)



with respect to C_U

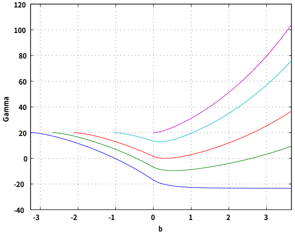


with respect to C_D

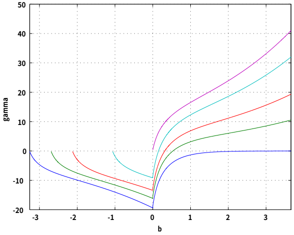
Linear Case

Suppose the running cost function is $f \equiv f_L$ where

$$f_L(x) := |x|, \quad x \in \mathbb{R}.$$

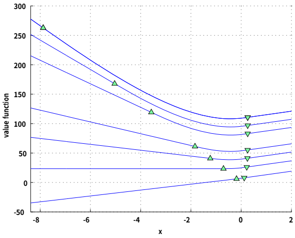


$\Gamma(a, \cdot)$

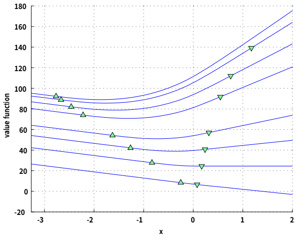


$\gamma(a, \cdot)$

Linear Case (Cont'd)



with respect to C_U



with respect to C_D

Inventory control with fixed costs

Inventory Models with Fixed Costs

1. Uncontrolled surplus: $X_t, t \geq 0$ defined on $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$.
2. An *(ordering) policy*

$$\pi := \left\{ U_t^\pi = \sum_{i: T_i^\pi \leq t} u_i^\pi; t \geq 0 \right\}$$

   in the form of an impulse control $(T_1^\pi, u_1^\pi; T_2^\pi, u_2^\pi; \cdots)$ where
   - $\{T_i; i \geq 1\}$ is an increasing sequence of $\mathbb{F}$-stopping times and
   - $u_i > 0$ is an $\mathcal{F}_{T_i}$-measurable random variable for $i \geq 1$.
3. Corresponding to every policy π, the (controlled) surplus process is

$$Y_t^\pi := X_t + U_t^\pi, \quad t \geq 0.$$

4. The problem is to minimize the total expected cost:

$$v_\pi(x) := \mathbb{E}_x \Big[\int_0^\infty e^{-qt} f(Y_t^\pi) \mathrm{d}t + \sum_{i=1}^\infty e^{-qT_i^\pi} (K + Cu_i^\pi) \Big].$$

The Model of X

- Compound Poisson models
- Brownian Motion models
- Sum of Compound Poisson and Brownian Motion
  - Bensoussan, Liu & Sethi (SICON, 2005), Benkherouf & Bensoussan (SICON, 2009).
- Spectrally negative Lévy models (this talk) – a general Lévy process with only positive jumps that is not a negative of a subordinator.

Assumptions

The assumptions are the same as those in Bensoussan, Liu & Sethi (SICON, 2005), Benkherouf & Bensoussan (SICON, 2009).

Assumption

$g(y) := Cy + K,\ y > 0$, for some unit cost of the item $C \in \mathbb{R}$ and fixed ordering cost $K > 0$.

Assumption

1. *f is a piecewise continuously differentiable function with $f(0) = 0$.*
2. *There exists a number a such that the function*

$$\tilde{f}(x) := f(x) + Cqx, \quad x \in \mathbb{R},$$

   is increasing on (a, ∞) and decreasing and convex on $(-\infty, a)$.
3. *There exist a $c_0 > 0$ and an $x_0 \geq a$ such that $\tilde{f}'(x) \geq c_0$ for $x \geq x_0$.*

The (s,S)-Policy

- For $-\infty < s < S < \infty$, an (s,S)-policy, $\pi_{s,S} := \{U_t^{s,S}; t \geq 0\}$, brings the level of the surplus process $Y^{s,S} := X + U^{s,S}$ up to S whenever it goes below s, with the corresponding NPV:

$$v_{s,S}(x) := \mathbb{E}_x\Big[\int_0^\infty e^{-qt} f(Y_t^{s,S})\mathrm{d}t + \sum_{0\leq t<\infty} e^{-qt} g(\Delta U_t^{s,S}) 1_{\{\Delta U_t^{s,S}>0\}}\Big].$$

- We aim to prove that the (s^*,S^*)-policy is optimal for some $-\infty < s^* < S^* < \infty$. Toward this end,
  - Write $v_{s,S}$ analytically using the scale function.
  - Choose the value of s^* and S^* using some smoothness condition.
  - Verify that the V_{s^*,S^*} satisfies QVI (quasi-variational inequality)

Computation of $v_{s,S}(x)$

1. We shall first rewrite

$$v_{s,S}(x) := \mathbb{E}_x\Big[\int_0^\infty e^{-qt} f(Y_t^{s,S})\mathrm{d}t + \sum_{0\le t<\infty} e^{-qt} g(\Delta U_t^{s,S}) 1_{\{\Delta U_t^{s,S}>0\}}\Big],$$

in terms of the scale function.

2. By the strong Markov property, it must satisfy,

$$v_{s,S}(x) = \mathbb{E}_x\Big[\int_0^{\tau_s^-} e^{-qt} f(X_t)\mathrm{d}t\Big] + \mathbb{E}_x\left[e^{-q\tau_s^-}(C(S - X_{\tau_s^-}) + K)\right] + \mathbb{E}_x\left[e^{-q\tau_s^-}\right] v_{s,S}(S).$$

3. Solving for $x = S$ gives $v_{s,S}(S)$ once we know these expectations.

Computation of $v_{s,S}(x)$

- Instead, we write

$$\tilde{v}_{s,S}(x) := v_{s,S}(x) + Cx$$
$$= k(s,x) + \left(1 - \frac{q}{\Phi(q)}\overline{\Theta}^{(q)}(x-s)\right)\tilde{v}_{s,S}(S), \quad x > s,$$

with $\overline{\Theta}^{(q)}(x) := W^{(q)}(x) - \Phi(q)\overline{W}^{(q)}(x)$ and

$$k(s,x) := \mathbb{E}_x\left[\int_0^{\tau_s^-} e^{-qt} f(X_t)\mathrm{d}t\right] - C\mathbb{E}_x\left[e^{-q\tau_s^-}X_{\tau_s^-}\right]$$
$$+ K\mathbb{E}_x\left[e^{-q\tau_s^-}\right] + Cx, \quad x > s.$$

- Then,

$$\tilde{v}_{s,S} := \tilde{v}_{s,S}(S) = \frac{\Phi(q)}{q}\frac{k(s,S)}{\overline{\Theta}^{(q)}(S-s)}, \quad S > s.$$

Computation of $v_{s,S}(x)$

- Define, for any measurable function h and $s \in \mathbb{R}$,

$$\Psi(s;h) := \int_0^\infty e^{-\Phi(q)y} h(y+s)\mathrm{d}y = \int_s^\infty e^{-\Phi(q)(y-s)} h(y)\mathrm{d}y,$$
$$\varphi_s(x;h) := \int_s^x W^{(q)}(x-y)h(y)\mathrm{d}y, \quad x \in \mathbb{R},$$
$$\mathcal{G}(s,x) := \Phi(q)\Psi(s;\tilde{f})\overline{W}^{(q)}(x-s) + K - \varphi_s(x;\tilde{f}), \quad x > s.$$

- For any $x > s$,

$$k(s,x) = \overline{\Theta}^{(q)}(x-s)\left[\Psi(s;\tilde{f}) - \frac{q}{\Phi(q)}\left(K + \frac{C\psi'(0+)}{q}\right)\right] + \mathcal{G}(s,x).$$

Choosing Candidates for (s, S)

To narrow down the candidates for (s, S), we shall choose these values so that

- the function $v_{s,S}$ is continuous/smooth enough and
- its slope at S is the same as the proportional cost C.

Lemma

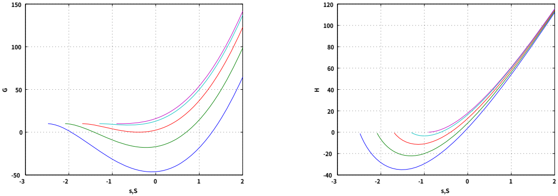
Suppose (s, S) *are such that* $\mathcal{G}(s, S) = \mathcal{H}(s, S) = 0$ *where*

$$\mathcal{H}(s, x) := \frac{\partial}{\partial x}\mathcal{G}(s, x).$$

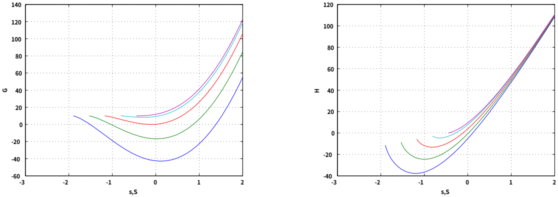
Then

1. $v_{s,S}$ *is continuous (resp. differentiable) at* s *when* X *is of bounded (resp. unbounded) variation,*
2. $\tilde{v}'_{s,S}(S) = 0$ *or equivalently* $v'_{s,S}(S) = C$.

Plots of $\mathcal{G}(s, S)$ and $\mathcal{H}(s, S)$



unbounded variation case



bounded variation case

Existence of (s^*, S^*)

1. Recall

$$\Psi(s;h) := \int_0^\infty e^{-\Phi(q)y} h(y+s)\mathrm{d}y = \int_s^\infty e^{-\Phi(q)(y-s)} h(y)\mathrm{d}y.$$

2. There exists a unique number a_0 such that $\Psi(a_0; \tilde{f}') = 0$, $\Psi(x; \tilde{f}') < 0$ if $x < a_0$ and $\Psi(x; \tilde{f}') > 0$ if $x > a_0$.
3. There exists $s^* < a_0$ and $S^* > a_0$ such that

$$s^* := \sup\left\{ s < a_0 : \inf_{S \geq s} \mathcal{G}(s,S) = 0 \right\} \quad \text{and} \quad S^* \in \arg\inf_{S \geq s} \mathcal{G}(s,S),$$

holds with $\mathcal{H}(s^*, S^*) = \mathcal{G}(s^*, S^*) = 0$.

Verification for Optimality

Proposition

1. $(\mathcal{L} - q)v_{s^*,S^*}(x) + f(x) = 0$ *for* $x > s^*$,
2. $(\mathcal{L} - q)v_{s^*,S^*}(x) + f(x) \geq 0$ *for* $x < s^*$.

Proposition

For every $x \in \mathbb{R}$, *we have* $v_{s^*,S^*}(x) \leq K + \inf_{u \geq 0} [Cu + v_{s^*,S^*}(x + u)]$.

Theorem

The (s^*, S^*)*-policy is optimal and the value function is given by*

$$v_{s^*,S^*}(x) = \frac{\Phi(q)}{q}\Psi(s^*; \tilde{f})Z^{(q)}(x - s^*) - \varphi_{s^*}(x; \tilde{f}) - \frac{C\psi'(0+)}{q} - Cx, \quad x >$$

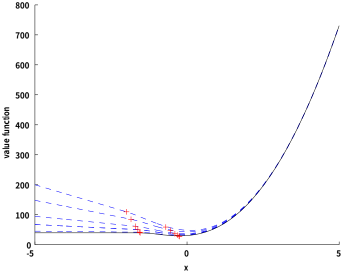
Numerical Examples

1. A spectrally negative Lévy process is said to be in the β-family if

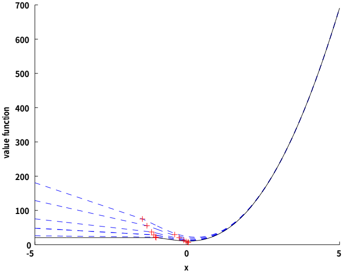
$$\psi(z) = \hat{\delta} z + \frac{1}{2}\sigma^2 z^2 + \frac{\varpi}{\beta}\left\{B(\alpha + \frac{z}{\beta}, 1-\lambda) - B(\alpha, 1-\lambda)\right\}$$

   for some $\hat{\delta} \in \mathbb{R}$, $\alpha > 0$, $\beta > 0$, $\varpi \geq 0$, $\lambda \in (0,3)\backslash\{1,2\}$ and the beta function $B(x,y) := \Gamma(x)\Gamma(y)/\Gamma(x+y)$.
2. We suppose $\hat{\delta} = 0.1$, $\lambda = 1.5$, $\alpha = 3$, $\beta = 1$ and $\varpi = 0.1$. With this specification, the process has jumps of infinite activity (and of bounded variation), which is not covered in the framework of Bensoussan, Liu & Sethi (SICON, 2005).
3. We consider $\sigma = 0$ and $\sigma = 0.2$ so as to study both the bounded and unbounded variation cases.
4. We let $q = 0.03$ and for the surplus cost we consider the quadratic case $f(x) = x^2$, $x \in \mathbb{R}$.

Sensitivity w.r.t. the Proportional Cost C



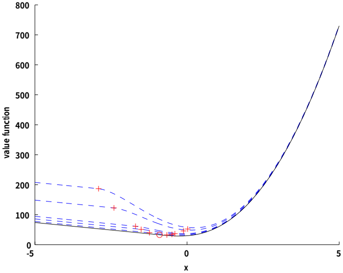
unbounded variation case ($\sigma > 0$)



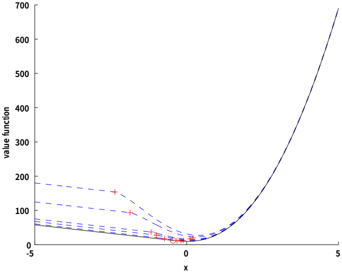
bounded variation case ($\sigma = 0$)

Figure : The value functions for various values of the proportional cost C.

Sensitivity w.r.t. the Fixed Cost K



unbounded variation case ($\sigma > 0$)



bounded variation case ($\sigma = 0$)

Figure : The value functions for various values of the fixed cost K.

References

[1] E. Baurdoux and K. Yamazaki. *Optimality of Doubly Reflected Levy Processes in Singular Control.* Stochastic Processes and their Applications, 125(7):2727-2751, 2015.

[2] K. Yamazaki. *Inventory Control for Spectrally Positive Lévy Demand Processes.* Mathematics of Operations Resarch, forthcoming.