Scenario tree reduction via quadratic process distances applied to hydrothermal scheduling problems

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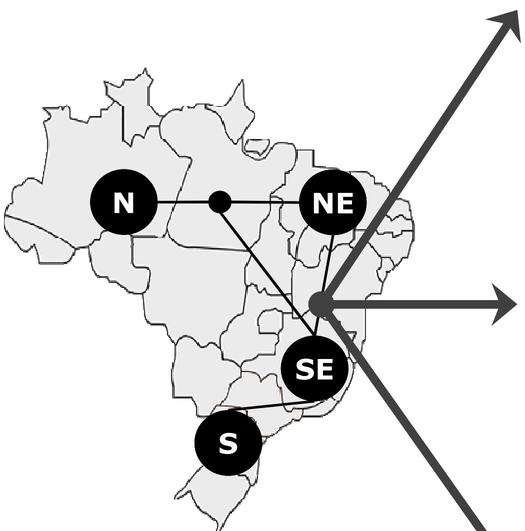
The Brazilian hydrothermal scheduling problem

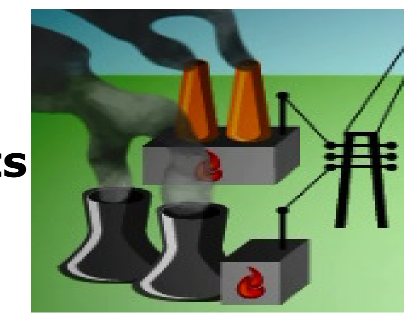
- Hydrothermal sistem



- **Hydro plants**

87.3 GW | **62 %**

- **Thermal plants**

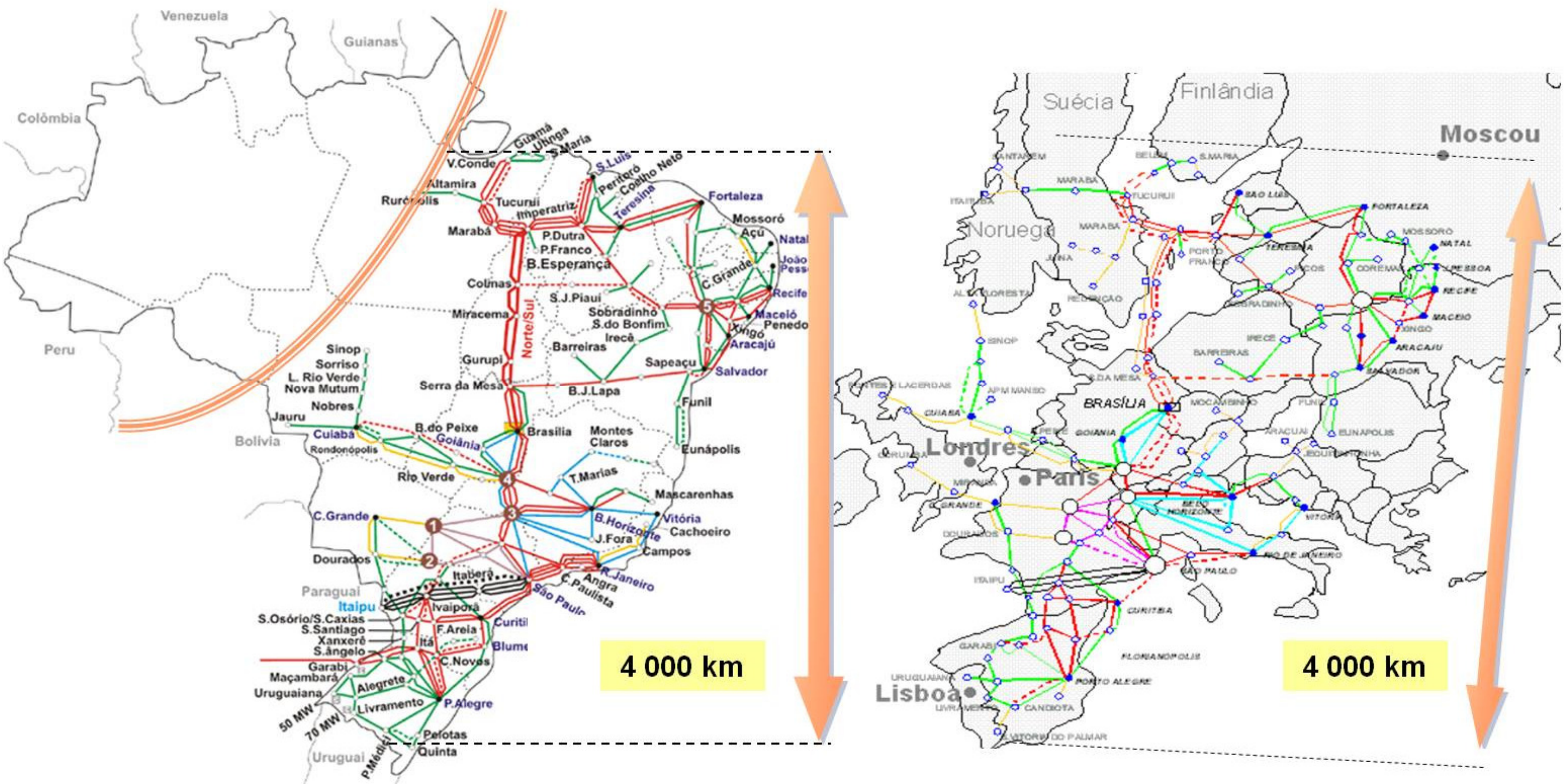


42.5 GW | **30 %**

- **Alternative sources**

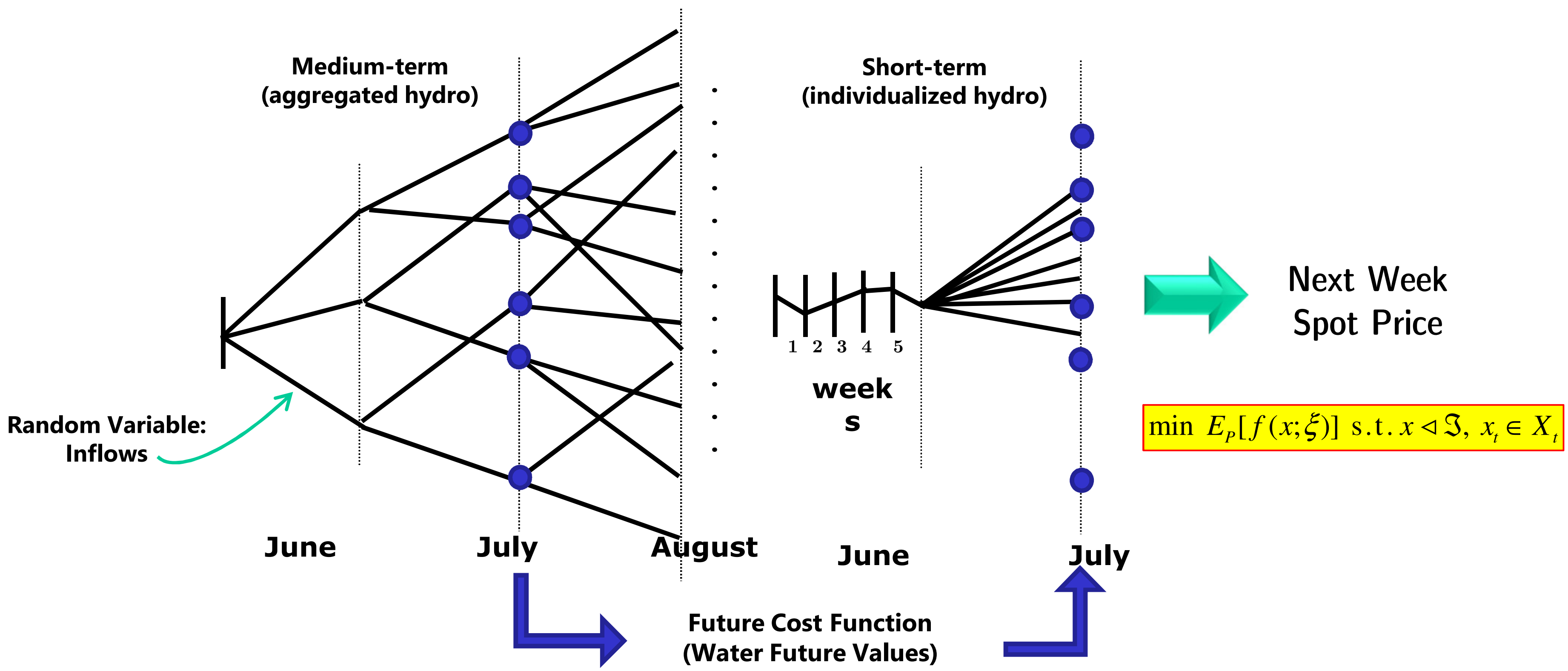
11.1 GW | **8 %**

The Brazilian hydrothermal scheduling problem



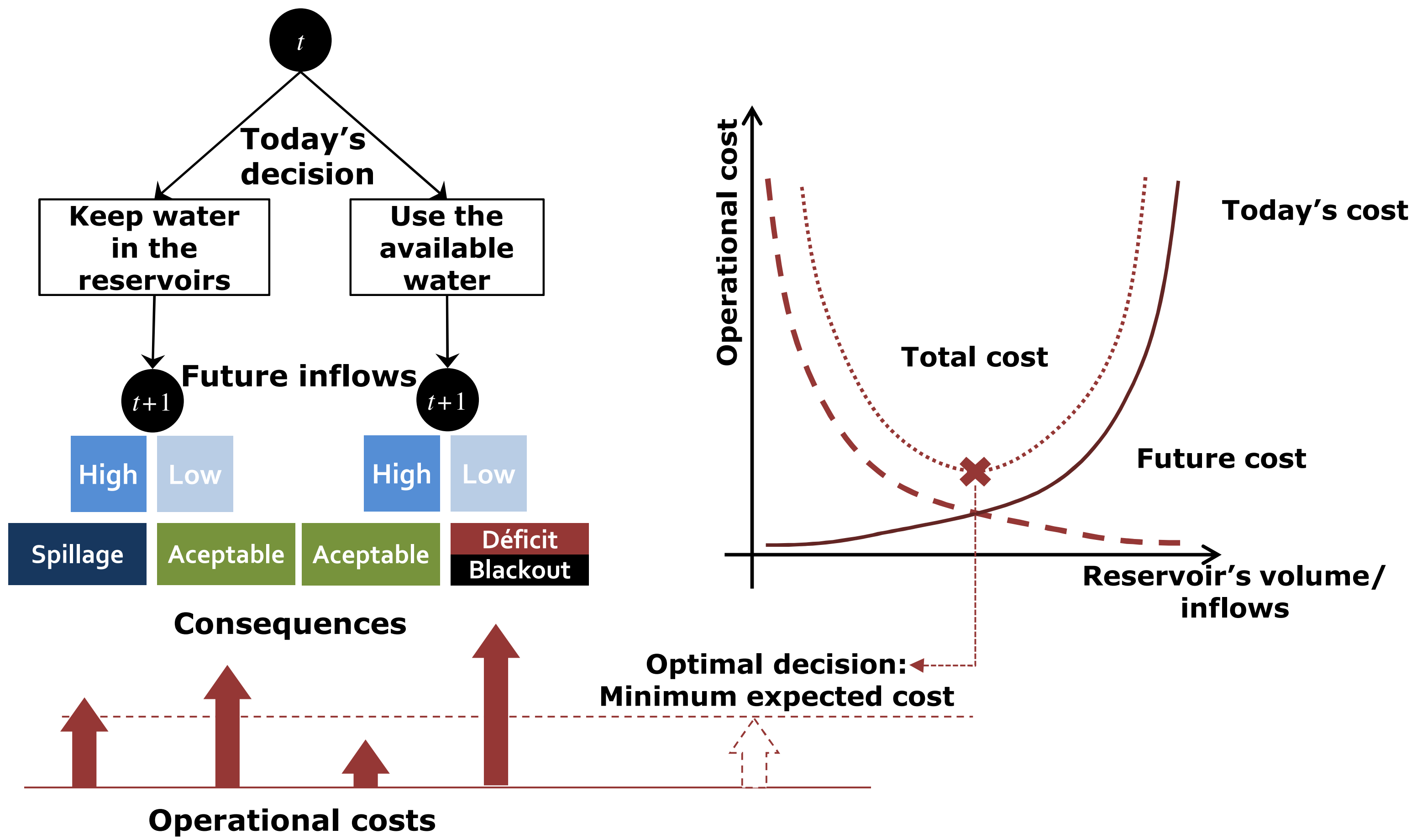
The Brazilian hydrothermal scheduling problem

- Due to the predominance of hydro resources, the Brazilian hydrothermal scheduling problem is solved by the following optimization models
  - Medium-term: 5-years planning horizon, with monthly steps
  - Short-term: 2-months planning horizon, with weekly steps in first month



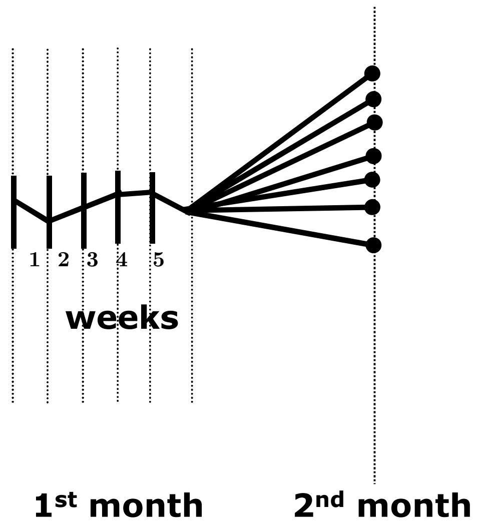
The Brazilian hydrothermal scheduling problem

- **The Brazilian HS problem is coupled in time**



The short-term HS problem – current approach

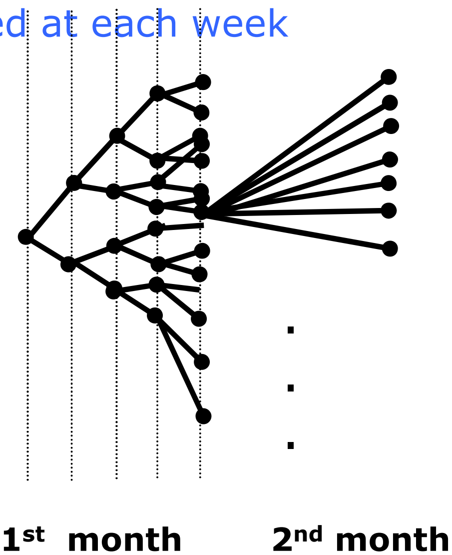
- Two-stage stochastic linear programming problem
  - The first month is split into weeks
  - Only one inflow scenario for each week (forecast)
  - Inflows are updated at each week
  - Weekly forecasting is a difficulty task. Differences between realized and forecasted values increase spot prices volatility



Two-stage programming problem

The short-term HS problem – new approach

- Multistage stochastic linear programming problem
  - The first month is split into weeks
  - Inflow scenarios are considered also in the weekly stages
  - Inflows are updated at each week



Multistage programming problem

> Scenario tree representativity

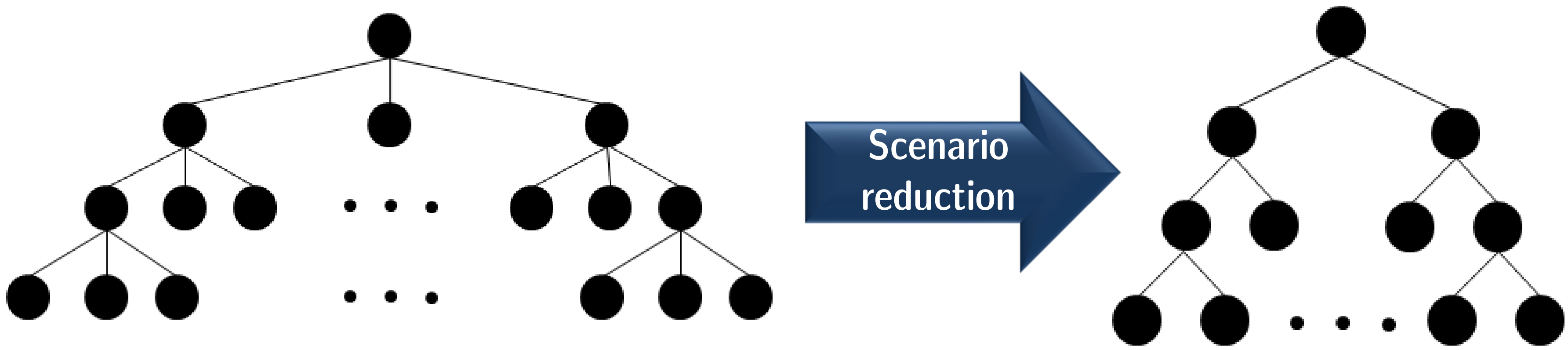
Improve the decisions

Increase computational burden

Challenge: to keep an equilibrium

Scenario tree reduction

- It is desirable to have:
  - **Large scenario trees to represent the stochastic process in a satisfactory manner**
  - **Small scenario trees to be able to solve the multistage stochastic program in a reasonable CPU time**



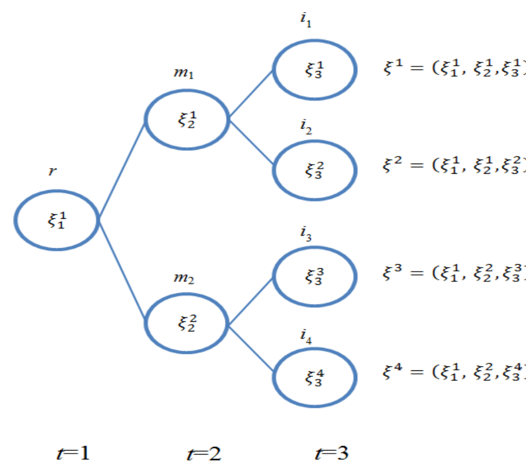
There are many alternatives to reduce scenario trees...

- Jardim et al. (2001)
  - **Based on clustering techniques (K-means)**
- Pflug (2001), Dupacova et al. (2003), Growe-Kuska et al. (2003)
  - **Based on probabilistic metrics**
  - **Stability results for two-stage stochastic programs**
- Heitsch and Romisch (2009), **Pflug (2009)**, Oliveira and Sagastizábal (2010), **Pflug and Pischler (2012)**, **Kovacevic and Pischler (2015)**
  - **Based on probabilistic metrics**
  - **Stability results for multistage stochastic programs**

Problem statement

- *T>0* is the planning horizon of the hydrothermal scheduling problem
- $\xi = (\xi_0, \ldots, \xi_T)$ is the stochastic process representing water inflows
- $\Xi = \Xi_0 \times \ldots \times \Xi_T$ support set, $\Im = (\Im_t)_{t=0}^{T}$ filtration

$$\min \; E_P[f(x;\xi)] \; \text{s.t.} \; x \lhd \Im, \; x_t \in X_t, \quad t = 0, \ldots, \mathrm{T}$$



- $f : R^n \times \Xi \to R$ is a linear function
- X_t is a polyhedral feasible set
- E_P is the expected value operator

Problem statement

- With two different filtered probability spaces

$$\boldsymbol{P} := \left(\Xi, \Im, P\right) \quad \text{and} \quad \boldsymbol{P}' := \left(\Xi, \Im', P'\right)$$

we associate two different stochastic programs:

$$v(\boldsymbol{P}) := \min \; E_P[f(x;\xi)] \text{ s.t. } x \lhd \Im, \; x_t \in X_t, \quad t = 0, \ldots, T$$

$$v(\boldsymbol{P}') := \min \; E_{P'}[f(x;\xi)] \text{ s.t. } x \lhd \Im', \; x_t \in X_t, \quad t = 0, \ldots, T$$

Nested distance

Pflug (2009), Pflug and Pischler (2012)

Definition *For two filtered probability spaces* $\boldsymbol{P} := (\Xi, \mathfrak{I}, P)$, $\boldsymbol{P}' := (\Xi, \mathfrak{I}', P')$ *and a real-valued distance function* $d : \Xi \times \Xi \to R_+$ *the process distance of order* $r \geq 1$ *is the optimal value of the following optimization problem*

$$D_r(\boldsymbol{P}, \boldsymbol{P}') := \begin{cases} \min_{\pi} & \left(\iint d(\xi, \xi')^r \pi(d\xi, d\xi') \right)^{\frac{1}{r}} \\ s.t. & \pi[A \times \Xi' \mid \mathfrak{I}_t \otimes \mathfrak{I}_t'] = P[A \mid \mathfrak{I}_t] \qquad (A \in \mathfrak{I}_T, \quad t = 0, \ldots, T) \\ & \pi[\Xi \times B \mid \mathfrak{I}_t \otimes \mathfrak{I}_t'] = P'[\mathrm{B} \mid \mathfrak{I}_t'] \qquad (B \in \mathfrak{I}_T', \quad t = 0, \ldots, T) \end{cases}$$

where the infimum is among all bivariate probability measures π on the product sigma algebra

$$\mathfrak{I}_T \otimes \mathfrak{I}_T' := \sigma\left(\{A \times B : A \in \mathfrak{I}_T, \; B \in \mathfrak{I}_T'\}\right).$$

Pflug and Pischler (2012) established that: $\left| v(\boldsymbol{P}) - v(\boldsymbol{P}') \right| \leq L_\beta D_r(\boldsymbol{P}, \boldsymbol{P}')^\beta.$

Quadratic process distance

Definition. (Quadratic process distance). *For all* $t = 0, \ldots, T$ *, let* M_t *be definite positive matrices and* $w_t > 0$ *given weights. The quadratic process distance is a particular case of the nested distance obtained by taking* $r = 2$ *and a distance function given by*

$$d(\xi, \xi') := \sum_{t=0}^{T} w_t \left\| \xi_t - \xi_t' \right\|_{M_t}^2 .$$

$$D_r(\boldsymbol{P}, \boldsymbol{P}') := \begin{cases} \min_{\pi} \left(\iint d(\xi, \xi')^r \pi(d\xi, d\xi') \right)^{\frac{1}{r}} \\ s.t. \quad \pi\left[A \times \Xi' \mid \mathfrak{I}_t \otimes \mathfrak{I}_t'\right] = P\left[A \mid \mathfrak{I}_t\right] \qquad (A \in \mathfrak{I}_T,\ t = 0, \ldots, T) \\ \qquad \pi\left[\Xi \times B \mid \mathfrak{I}_t \otimes \mathfrak{I}_t'\right] = P'\left[\mathrm{B} \mid \mathfrak{I}_t'\right] \qquad (B \in \mathfrak{I}_T',\ t = 0, \ldots, T) \end{cases}$$

$$\|x\|_M = \sqrt{x^T M x}$$

Nested distance X Wasserstein distance

Nested distance:

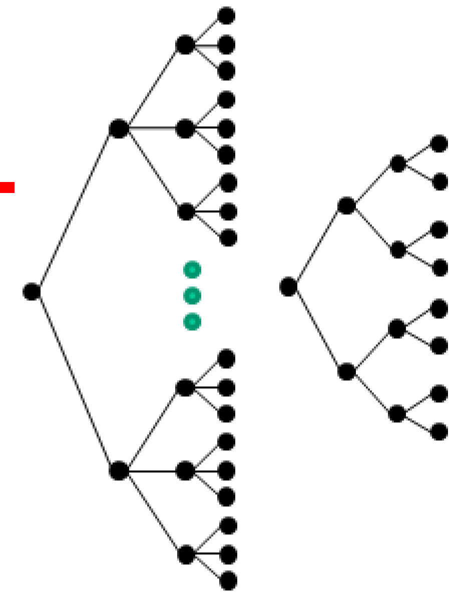
$$D_r(\boldsymbol{P},\boldsymbol{P}') := \begin{cases} \min\limits_{\pi} & \left(\iint d(\xi,\xi')^r \pi(d\xi,d\xi')\right)^{\frac{1}{r}} \\ s.t. & \pi\left[A\times\Xi' \mid \mathfrak{I}_t \otimes \mathfrak{I}_t'\right] = P\left[A \mid \mathfrak{I}_t\right] \qquad (A\in\mathfrak{I}_T,\quad t=0,\ldots,T) \\ & \pi\left[\Xi\times B \mid \mathfrak{I}_t \otimes \mathfrak{I}_t'\right] = P'\left[\mathrm{B} \mid \mathfrak{I}_t'\right] \qquad (B\in\mathfrak{I}_T',\ t=0,\ldots,T) \end{cases}$$

Wasserstein distance:

$$d_r(\boldsymbol{P},\boldsymbol{P}') := \begin{cases} \min\limits_{\pi} & \left(\iint d(\xi,\xi')^r \pi(d\xi,d\xi')\right)^{\frac{1}{r}} \\ s.t. & \pi\left[A\times\Xi'\right] = P\left[A\right] \qquad (A\in\mathfrak{I}_T) \\ & \pi\left[\Xi\times B\right] = P'\left[\mathrm{B}\right] \qquad (B\in\mathfrak{I}_T') \end{cases}$$

The inequality $d_r(\boldsymbol{P},\ \boldsymbol{P}') \le D_r(\boldsymbol{P},\ \boldsymbol{P}')$ always holds.

Quadratic process distance for scenario trees



$$
D_2(\boldsymbol{P},\boldsymbol{P}') = \begin{cases} \min\limits_{\pi} \sum\limits_{i\in\aleph_T} \sum\limits_{j\in\aleph'_T} d(\xi^i,\xi'^j)\pi_{ij} \\ s.t. \quad \sum\limits_{j\in\aleph'_T:n\Supset j} \pi_{ij} = \dfrac{P(i)}{P(m)} \sum\limits_{i'\in\aleph_T:\,m\,\Supset\, i'} \; \sum\limits_{j'\in\aleph'_T:\,n\,\Supset\, j'} \pi_{i'j'} \qquad (m \Supset i,\ n) \\ \qquad \sum\limits_{i\in\aleph_T:m\Supset i} \pi_{ij} = \dfrac{P'(j)}{P'(n)} \sum\limits_{i'\in\aleph_T:\,m\,\Supset\, i'} \; \sum\limits_{j'\in\aleph'_T:\,n\,\Supset\, j'} \pi_{i'j'} \qquad (n \Supset j,\ m) \\ \qquad \pi_{ij} \geq 0 \ \text{ and } \ \sum\limits_{i,j} \pi_{ij} = 1. \end{cases}
$$

- Suppose that scenario trees are employed to model the process ξ and the related filtration.
- The set of nodes of a given stage t is $\aleph_t$.
- The direct predecessor of a node $n \in \aleph$ is represented by n_- and its successors (children) form a set of nodes denoted by n_+.
- We employ the notation $m \Supset i$ (or equivalently $i \Subset m$) to mean that an intermediate node m is a predecessor of the leaf node i (no matter the stage t).
- The probability measure for the nested distance is given by masses π_{ij} at the leaves $i \in \aleph_T$ and $j \in \aleph'_T$

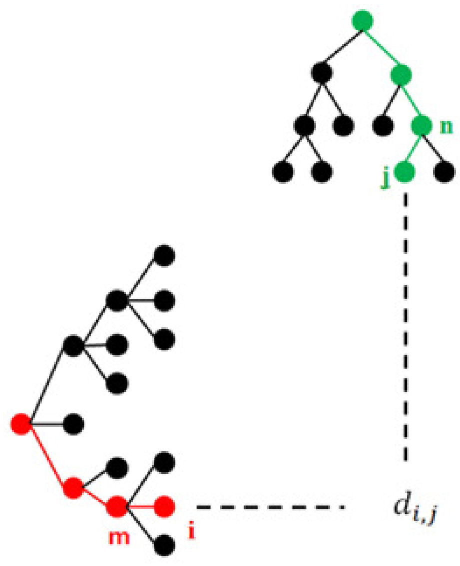
Quadratic process distance for scenario trees

$$D_2(\boldsymbol{P},\boldsymbol{P}') = \begin{cases} \min_{\pi} \sum_{i\in\aleph_T} \sum_{j\in\aleph'_T} d(\xi^i, \xi'^j)\pi_{ij} \\ s.t. \quad \sum_{j\in\aleph'_T : n \ni j} \pi_{ij} = \frac{P(i)}{P(m)} \sum_{i'\in\aleph_T : m \ni i'} \sum_{j'\in\aleph_T : n \ni j'} \pi_{i'j'} \quad (m \ni i,\ n) \\ \quad \sum_{i\in\aleph_T : m \ni i} \pi_{ij} = \frac{P'(j)}{P'(n)} \sum_{i'\in\aleph_T : m \ni i'} \sum_{j'\in\aleph_T : n \ni j'} \pi_{i'j'} \quad (n \ni j,\ m) \\ \quad \pi_{ij} \geq 0 \text{ and } \sum_{i,j} \pi_{ij} = 1. \end{cases}$$

P(i), P(m), P′(j) and P′(n) are given probabilities (of nodes)

The nested (quadratic) distance between two trees is a LP!

Pflug and Pischler (2012) showed that this LP can be decomposed into pairs of nodes



$$\begin{cases} \min_{\eta} \sum_{i\in m_+} \sum_{j\in n_+} D(i,j)\,\eta_{ij} \\ s.t. \quad \sum_{j\in n_+} \eta_{ij} = P(i/m) \quad (i\in m_+) \\ \quad \sum_{i\in m_+} \eta_{ij} = P'(j/n) \quad (j\in n_+) \\ \quad \eta \geq 0\ . \end{cases}$$

Quadratic process distance for scenario trees

$$D_2(\boldsymbol{P},\boldsymbol{P}')=\begin{cases}\min\limits_{\pi} \sum\limits_{i\in\aleph_T}\sum\limits_{j\in\aleph_T'} d(\xi^i,\xi'^j)\pi_{ij} \\ s.t. \quad \sum\limits_{j\in\aleph_T':n\ni j}\pi_{ij}=\dfrac{P(i)}{P(m)}\sum\limits_{i'\in\aleph_T:m\ni i'}\ \sum\limits_{j'\in\aleph_T:n\ni j'}\pi_{i'j'} \qquad (m\ni i,\ n) \\ \quad\ \sum\limits_{i\in\aleph_T:m\ni i}\pi_{ij}=\dfrac{P'(j)}{P'(n)}\sum\limits_{i'\in\aleph_T:m\ni i'}\ \sum\limits_{j'\in\aleph_T:n\ni j'}\pi_{i'j'} \qquad (n\ni j,\ m) \\ \quad\ \pi_{ij}\geq 0 \ \text{ and } \sum\limits_{i,j}\pi_{ij}=1.\end{cases}$$

Pflug and Pischler (2012) show that this LP can be split into pairs of nodes

$$D(i,j):=d(\xi^i,\xi'^j) \text{ for all } i\in\aleph_T \text{ and } j\in\aleph_T'$$

$$D(m,n):=\sum_{i\in m_+}\sum_{j\in n_+}\pi^*(i,j/m,n)D(i,j) \text{ for all } m\in\aleph_t \text{ and } n\in\aleph_t',\ t=0,\dots,T,$$

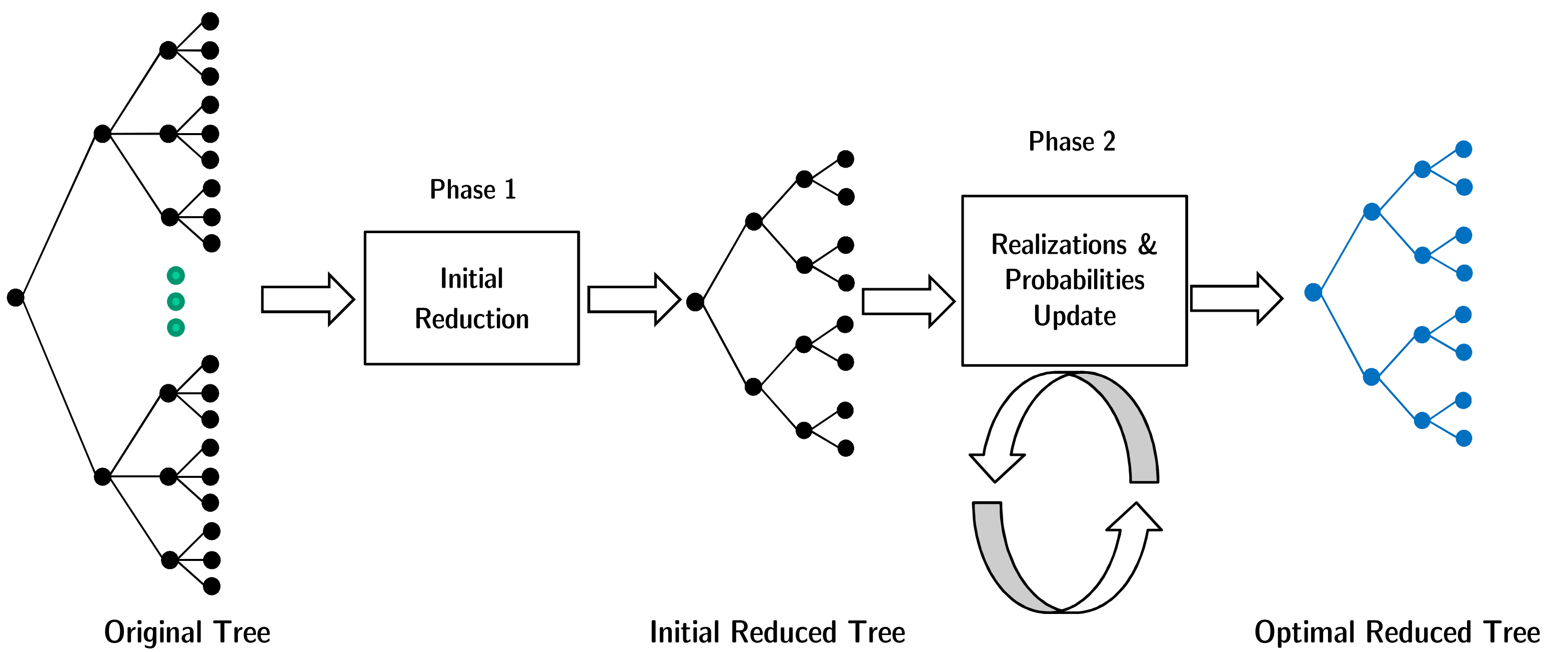
$$\begin{cases}\min\limits_{\eta} \sum\limits_{i\in m_+}\sum\limits_{j\in n_+} D(i,j)\,\eta_{ij} \\ s.t. \quad \sum\limits_{j\in n_+}\eta_{ij}=P(i/m) \qquad (i\in m_+) \\ \quad\ \sum\limits_{i\in m_+}\eta_{ij}=P'(j/n) \qquad (j\in n_+) \\ \quad\ \eta\geq 0\,.\end{cases}$$

$$\pi^*(i,j/\mathrm{m},\mathrm{n})=\eta^*_{ij}$$

$$\pi^*_{ij}=\pi^*(i,j/i_{T-1},j_{T-1})\pi^*(i_{T-1},j_{T-1}/i_{T-2},j_{T-2})\cdots\pi^*(i_1,j_1/0,0)$$

Scenario reduction algorithm (Kovacevic & Pichler, 2015)

- Employ the nested distance as main tool



Scenario reduction

The algorithm performs as follows:

(i) given an approximating tree, find probabilities that decrease the nested distance

This can be done by solving a sequence of LPs

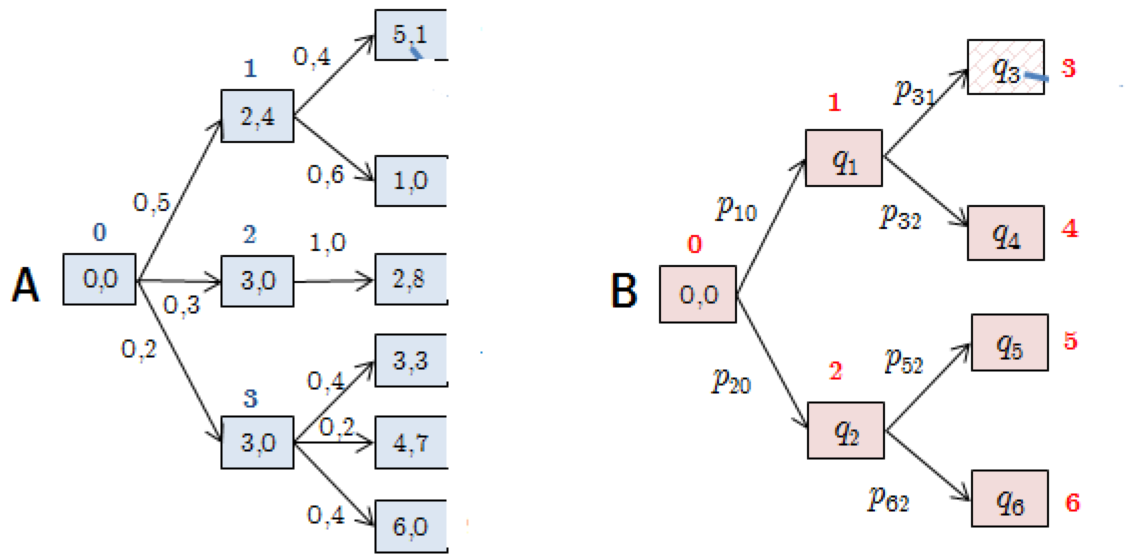
(ii) given the probabilities, update the node values of the approximating tree in such a way that the approximation is improved

There exists a closed expression in the case of the Quadratic process distance

Scenario reduction

(i) given an approximating tree, find probabilities that decrease the process distance

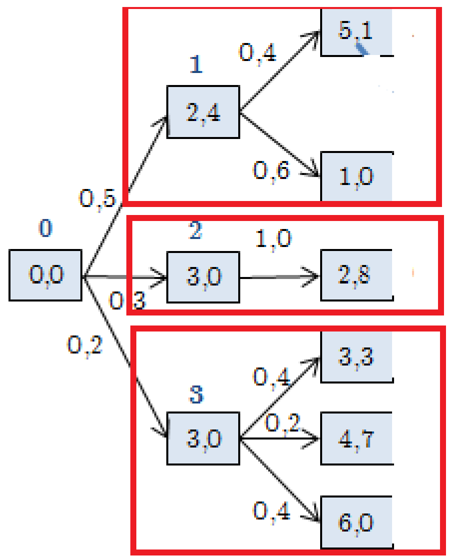
$$\begin{cases} \min_{\eta, P'(\cdot/n)} \sum_{m \in \aleph_t} \left(\sum_{i \in m_+} \sum_{j \in n_+} D(i,j)\, \eta_{ij} \right) \\ s.t. \quad \sum_{j \in n_+} \eta_{ij} = P(i/m) \qquad (i \in m_+) \\ \qquad \sum_{i \in m_+} \eta_{ij} = P'(j/n) \qquad (j \in n_+ \text{ and for all } m \in \aleph_t) \\ \qquad \boxed{\eta \geq 0, \;\; P'(\cdot/n) \geq 0.} \end{cases}$$



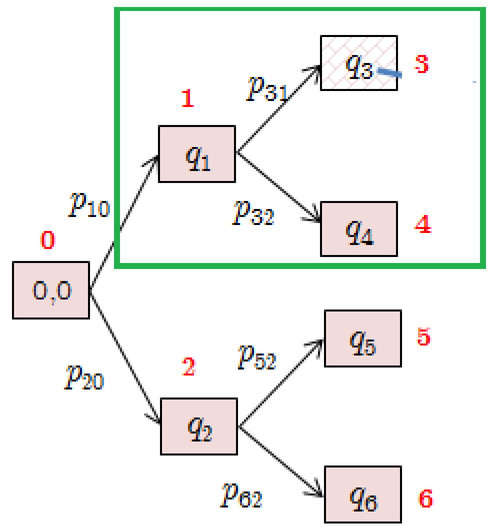
A
0
0,0
0,5
0,3
0,2
1
2,4
2
3,0
3
3,0
0,4
0,6
1,0
0,4
0,2
0,4
5,1
1,0
2,8
3,3
4,7
6,0
B
0
0,0
p_{10}
p_{20}
1
q_1
2
q_2
p_{31}
p_{32}
p_{52}
p_{62}
q_3
3
q_4
4
q_5
5
q_6
6

$$\begin{cases} \min\limits_{\eta, P'(\cdot/n)} \sum\limits_{m\in\aleph_t} \left(\sum\limits_{i\in m_+}\sum\limits_{j\in n_+} D(i,j)\,\eta_{ij}\right) \\ s.t. \ \sum\limits_{j\in n_+}\eta_{ij} = P(i/m) \qquad (i\in m_+) \\ \qquad \sum\limits_{i\in m_+}\eta_{ij} = P'(j/n) \qquad (j\in n_+ \text{ and for all } m\in\aleph_t) \\ \qquad \eta\ge 0,\ \ P'(\cdot/n)\ge 0. \end{cases}$$

A

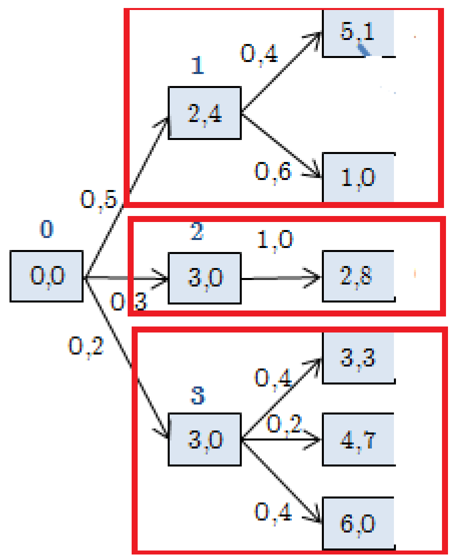


B

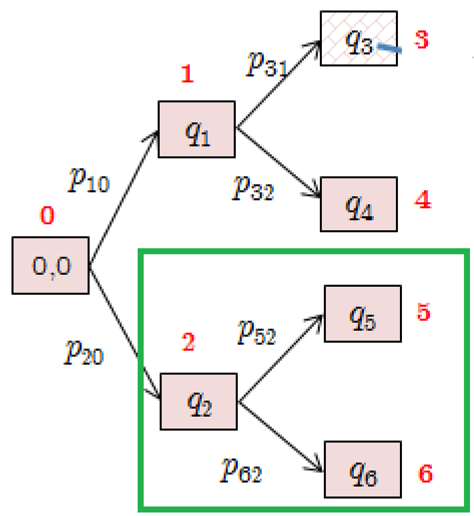


$$
\begin{cases}
\min\limits_{\eta, P'(\cdot/n)} \sum\limits_{m \in \aleph_t} \left(\sum\limits_{i \in m_+} \sum\limits_{j \in n_+} D(i,j)\, \eta_{ij} \right) \\
s.t. \quad \sum\limits_{j \in n_+} \eta_{ij} = P(i/m) \qquad (i \in m_+) \\
\qquad \sum\limits_{i \in m_+} \eta_{ij} = P'(j/n) \qquad (j \in n_+ \text{ and for all } m \in \aleph_t) \\
\qquad \eta \geq 0, \quad P'(\cdot/n) \geq 0.
\end{cases}
$$

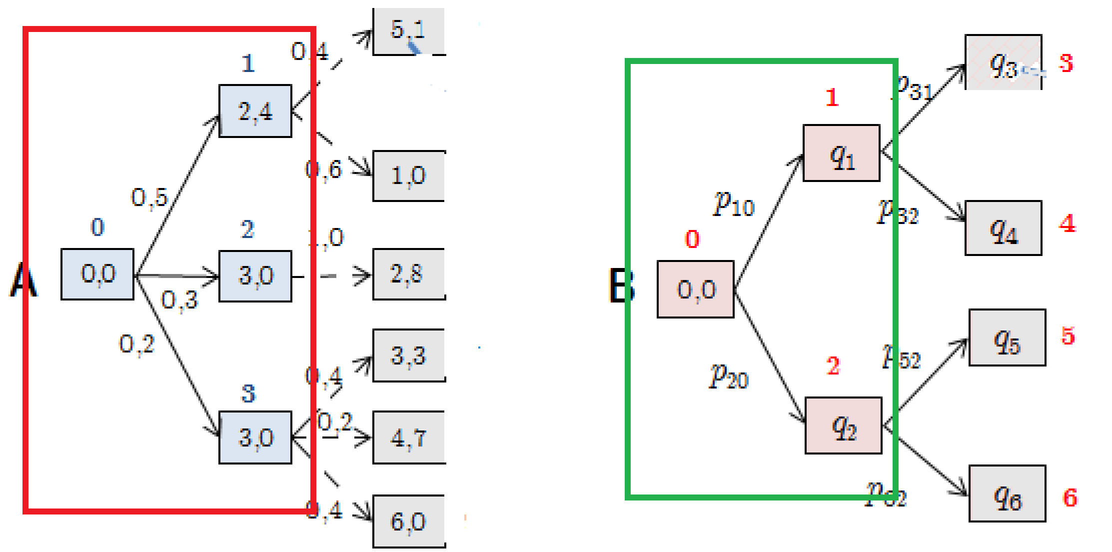
A



B



$$\begin{cases} \min\limits_{\eta, P'(\cdot/n)} \sum\limits_{m\in\aleph_t} \left(\sum\limits_{i\in m_+}\sum\limits_{j\in n_+} D(i,j)\,\eta_{ij}\right) \\ s.t. \quad \sum\limits_{j\in n_+} \eta_{ij} = P(i/m) \qquad (i\in m_+) \\ \qquad \sum\limits_{i\in m_+}\eta_{ij} = P'(j/n) \qquad (j\in n_+ \text{ and for all } m\in\aleph_t) \\ \qquad \eta\ge 0,\ \ P'(\cdot/n)\ge 0. \end{cases}$$



Scenario reduction

(ii) given the probabilities, update the node values of the approximating tree in such a way that the approximation is improved

$$\min_q \sum_{i \in \aleph_T} \sum_{j \in \aleph_T'} \pi_{ij} d(\xi^i, q^j)$$

The particular distance function $d(\xi^i; q^j) := \sum_{t=0}^{T} w_t \left\| \xi_t^i - q_t^j \right\|_{M_t}^2$ **yields a closed expression for the optimal solution**

$$(q_t^{n_t})^{k+1} = \sum_{m_t \in \mathcal{N}_t} \frac{\pi^k(m_t, n_t)}{\sum_{m_t \in \mathcal{N}_t} \pi^k(m_t, n_t)} \cdot \xi_t^{m_t},$$

$$\|x\|_M = \sqrt{x^T M x}$$

Scenario reduction

$$\min_q \sum_{i \in \aleph_T} \sum_{j \in \aleph_T'} \pi_{ij} d(\xi^i, \mathrm{q}^j)$$

$$d(\xi^i; \mathrm{q}^j) := \sum_{t=0}^{T} w_t \left\| \xi_t^i - q_t^j \right\|_{M_t}^2$$

$$(q_t^{n_t})^{k+1} = \sum_{m_t \in \mathcal{N}_t} \frac{\pi^k(m_t, n_t)}{\sum_{m_t \in \mathcal{N}_t} \pi^k(m_t, n_t)} \cdot \xi_t^{m_t},$$

Kovacevic and Pischler (2012) use the matrix M = I (identity)

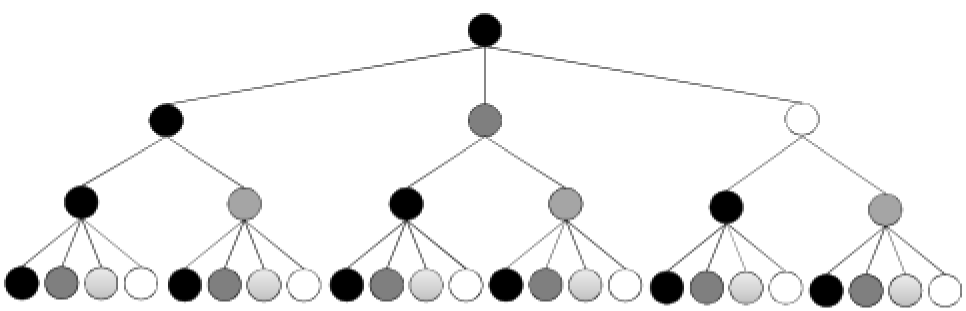
We propose to use information from the problem to define other matrices:

- **M = inverse of the correlation matrix (Mahalanobis distance)**
- **M = power capacity of hydro power plants**

$$\|x\|_M = \sqrt{x^T M x}$$

Scenario tree generation
The HS problem

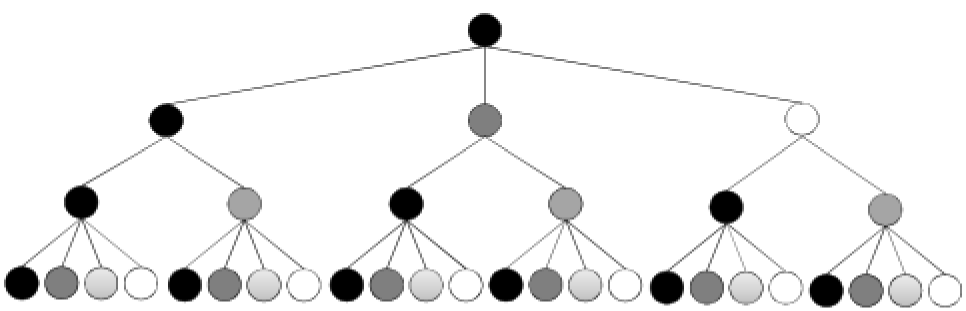
- A large (original) scenario tree is generated considering a stage-wise independent model
- Water inflow in each hydroelectric reservoir follows a three-parameter Lognormal distribution
- Water inflows are correlated

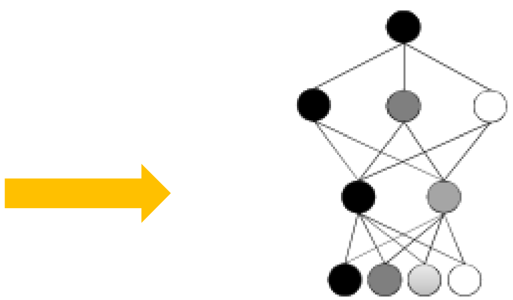


Each node is a vector with inflows of many hydro plants

Scenario tree generation
The HS problem

- A large (original) scenario tree is generated considering stage-wise independent model
- Water inflow in each hydroelectric reservoir follows a three-parameter Lognormal distribution
- Water inflows are correlated





This assumption simplifies the scenario tree reduction algorithm!

Assessing quality of the reduced trees

- Stochastic program based on the large (original) scenario tree

$$v(\boldsymbol{P}) := \min\ E_P[f(x;\xi)]\ \text{s.t.}\ x \lhd \Im,\ x_t \in X_t,\ \ t = 0,\ldots,T \Rightarrow \boxed{x^{P}}$$

- Stochastic program based on the reduced scenario tree

$$v(\boldsymbol{P}') := \min\ E_{P'}[f(x;\xi)]\ \text{s.t.}\ x \lhd \Im',\ x_t \in X_t,\ \ t = 0,\ldots,T \Rightarrow \boxed{x^{P'}}$$

We solve both problems by the Nested Decomposition

Assessing quality of the reduced trees

$$v(P) := \min \ E_P[f(x;\xi)] \ \text{s.t.} \ x \lhd \Im, \ x_t \in X_t, \quad t = 0,\ldots,T$$

- $\min_{\substack{A_1 x_1 = b_1 \\ x_1 \geq 0}} c_1^\top x_1 + \mathcal{Q}_2(x_2, \xi_{[2]})$
- $\mathcal{Q}_{t+1}(x_t, \xi_{[t]}) = \mathbb{E}_{|\xi_{[t]}}[Q_{t+1}(x_t, \xi_{[t+1]})]$ for $t = 1, \ldots, T-1$, e $\mathcal{Q}_{T+1}(x_T, \xi_{[T]}) = 0$
- $Q_t(x_{t-1}, \xi_{[t]}) = \min_{x_t \geq 0} \ c_t^\top x_t + \mathcal{Q}_{t+1}(x_t, \xi_{[t]})$ s.t $B_t x_{t-1} + A_t x_t = b_t$.

Cutting-plane approximation

$$v(P) := \min \; E_P[f(x;\xi)] \;\; \text{s.t.}\; x \lhd \Im,\; x_t \in X_t,\;\; t = 0,\ldots,T$$

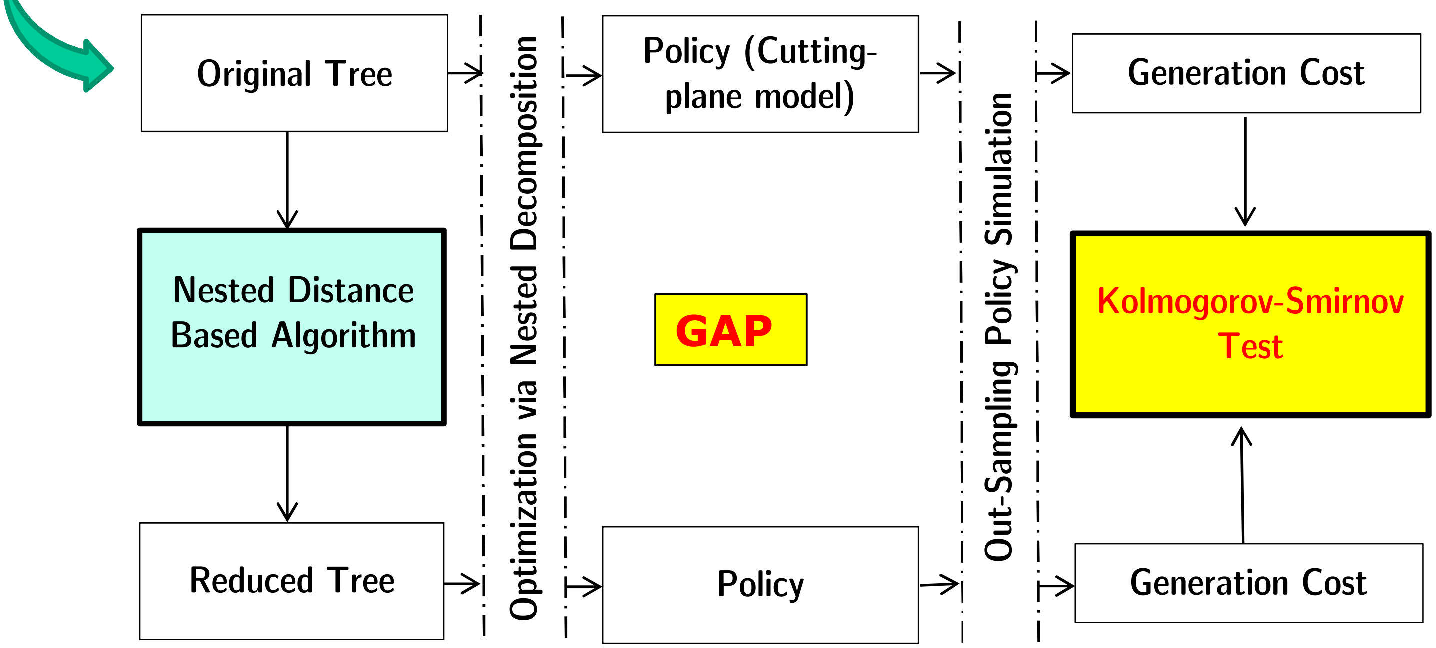
- $\min_{\substack{A_1 x_1 = b_1 \\ x_1 \geq 0}} c_1^\top x_1 + \check{\mathcal{Q}}_2(x_2, \xi_{[2]})$
- $\check{\mathcal{Q}}_{t+1}(x_t, \xi_{[t]}) = \mathbb{E}_{|\xi_{[t]}}[\underline{Q_{t+1}}(x_t, \xi_{[t+1]})]$ for $t = 1, \ldots, T-1$, e $\check{\mathcal{Q}}_{T+1}(x_T, \xi_{[T]}) = 0$
- $\underline{Q_t}(x_{t-1}, \xi_{[t]}) = \min_{x_t \geq 0} \; c_t^\top x_t + \mathcal{Q}_{t+1}(x_t, \xi_{[t]})$ s.t $B_t x_{t-1} + A_t x_t = b_t$.

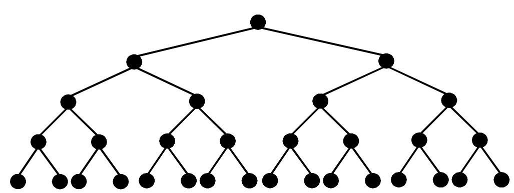
$$v(P') := \min \; E_{P'}[f(x;\xi)] \;\; \text{s.t.}\; x \lhd \Im',\; x_t \in X_t,\;\; t = 0,\ldots,T \;\; \Rightarrow \;\; \boxed{\mathbf{x}^{\mathbf{P'}}}$$

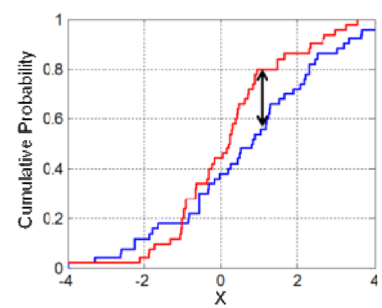
$$\boxed{\textbf{Gap} = \quad c_1^\top \mathbf{x}^{\mathbf{P'}} + \check{\mathcal{Q}}_2(\mathbf{x}^{\mathbf{P'}} \xi_{[2]}) \quad - \quad v(P)}$$

General Framework

A stage-wise independent scenario tree is obtained considering an inflow time-independent model (three-parameter Lognormal distribution)







Numerical Experiments

- ## Thermal plants data

Termo	c	Pmax
1	10	350
2	20	300
3	40	275
4	70	200
5	100	150

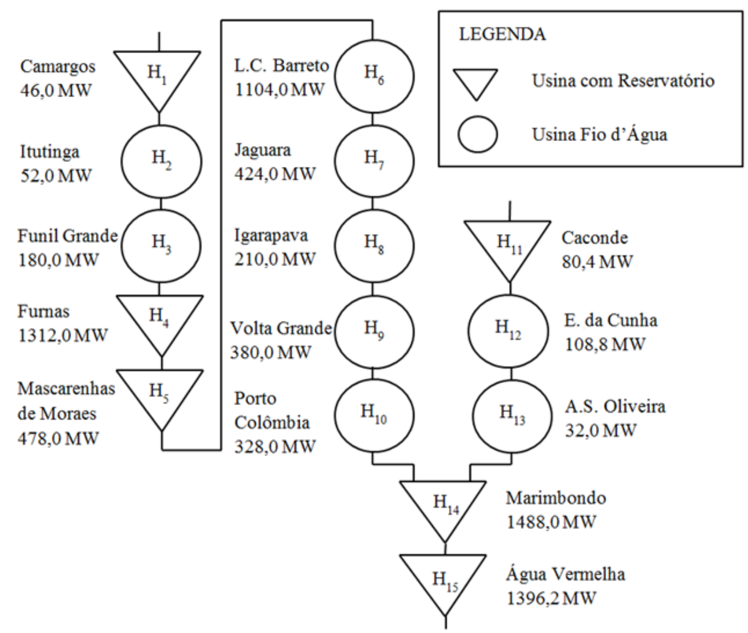
- ## Load

Load [MWmês]	7000

- ## Hydro plants data

Hidro	Vin (%)	Qmax (m^3/seg)	Vmin (hm^3)	Vmax (hm^3)	Produt (MW/m^3)
1	50	220	120	792	0,1783
2	0	236	0	0	0,2447
3	0	585	0	0	0,3457
4	50	1688	5733	22950	0,7475
5	0	1040	0	0	0,3160
6	0	2028	0	0	0,5627
7	0	1076	0	0	0,4043
8	0	1480	0	0	0,1525
9	0	1584	0	0	0,2472
10	0	1988	0	0	0,2038
11	50	141	51	555	0,7754
12	0	148	0	0	0,7461
13	0	96	0	0	0,2064
14	50	2944	890	6150	0,4663
15	50	2958	5856	11025	0,4568

- ## Cascade



Numerical Results - I

$$d(\xi,\xi') := \sum_{t=0}^{T} w_t \left\| \xi_t - \xi_t' \right\|_{M_t}^2$$

OT: Original Tree, RT: Reduced Tree, K-S Test is approved if < 0.043

Case	Structure of the Tree	Scenario Distance Matrix (M)	Out-sampling Policy Simulation (2,000 scenarios) Operating Cost (R$)			Computational Time (minutes)		
			Expected cost	GAP (%)	K-S Test	Nested Distance	Nested Decomposition	Total
OT	1*5*5*5*5: 625	-	430,248	-	-		99.89	99.89
RT1	1*2*5*5*5: 250	Identity	429,611	0.15	**0.0195**	0.015	13.08	13.10
		Plant Capacity	429,611	0.15	**0.0195**	0.017	13.15	13.17
		Inverse Correlation	429,640	0.14	**0.0190**	0.011	16.09	16.10
RT2	1*2*2*2*5: 40	Identity	432,560	0.54	**0.0385**	0.016	2.19	2.21
		Plant Capacity	432,518	0.53	**0.0350**	0.017	2.16	2.18
		Inverse Correlation	430,813	0.13	**0.0240**	0.024	2.16	2.18
RT3	1*5*2*2*2: 40	Identity	437,282	1.63	**0.0285**	0.017	2.33	2.35
		Plant Capacity	436,816	1.53	**0.0290**	0.015	2.37	2.39
		Correlation Inverse	434,517	0.99	**0.0195**	0.024	2.41	2.43

Numerical Results - II

OT: Original Tree, RT: Reduced Tree, K-S Test is approved if < 0.043

Case	Structure of the Tree	Scenario Distance Matrix (M)	Out-sampling Policy Simulation (2,000 scenarios) Operating Cost (R$)			Computational Time (minutes)		
			Expected cost	GAP (%)	K-S Test	Nested Distance	Nested Decomposition	Total
OT	1*10*10*10: 1000	-	82,853	-	-	-	397.43	397.43
RT1	1*3*5*8: 120	Identity	82,551	1.27	**0.0385**	0.025	7.21	7.24
		Plant Capacity	83,140	0.90	**0.0380**	0.025	7.37	7.40
		Inverse Correlation	84,488	1.62	**0.0170**	0.017	7.32	7.34
RT2	1*8*5*3: 120	Identity	95,603	16.85	**0.1005**	0.027	9.27	9.30
		Plant Capacity	89,592	16.85	**0.0755**	0.027	9.40	9.43
		Correlation Inverse	82,853	9.50	**0.0650**	0.021	9.33	9.35

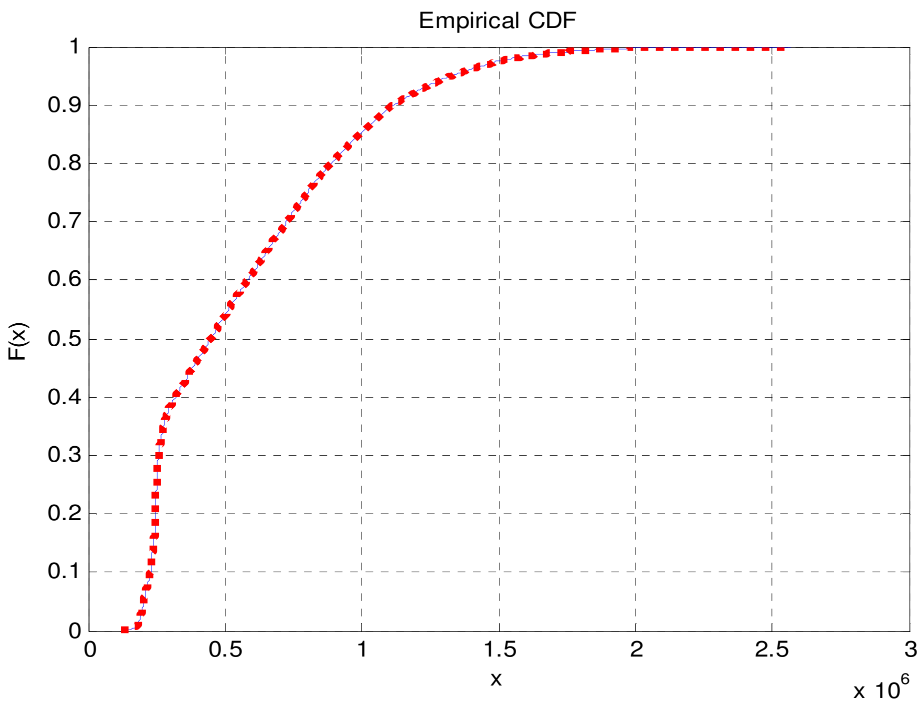
Numerical Results - III

OT: Original Tree, RT: Reduced Tree, K-S Test is approved if < 0.019

Case	Structure of the Tree	Scenario Distance Matrix (M)	Out-sampling Policy Simulation (10,000 scenarios) Operating Cost (R$)			Computational Time (minutes)		
			Expected cost	GAP (%)	K-S Test	Nested Distance	Nested Decomposition	Total
OT	1*4*4*4*4*4*4: 4096	-	572,183	-	-	-	>1,000	>1,000
RT1	1*2*2*4*4*4*4: 1024	Identity	572,197	0.00	**0.0031**	0.006	288.65	288.66
		Plant Capacity	572,433	0.04	**0.0027**	0.01	183.36	183.37
		Inverse Correlation	572,164	0.00	**0.0027**	0.01	319.14	319.15
RT2	1*2*2*2*4*4*4: 512	Identity	572,003	0.03	**0.0381**	0.01	26.70	26.71
		Plant Capacity	572,164	0.00	**0.0027**	0.01	11.92	11.93
		Inverse Correlation	572,660	0.08	**0.0389**	0.01	26.82	26.83
RT3	1*2*2*2*4*4*4: 256	Identity	575,500	0.58	**0.0518**	0.02	15.00	15.02
		Plant Capacity	575,351	0.55	**0.0515**	0.02	14.80	14.82
		Correlation Inverse	574,707	0.44	**0.0520**	0.01	3.80	3.81

First stage decisions

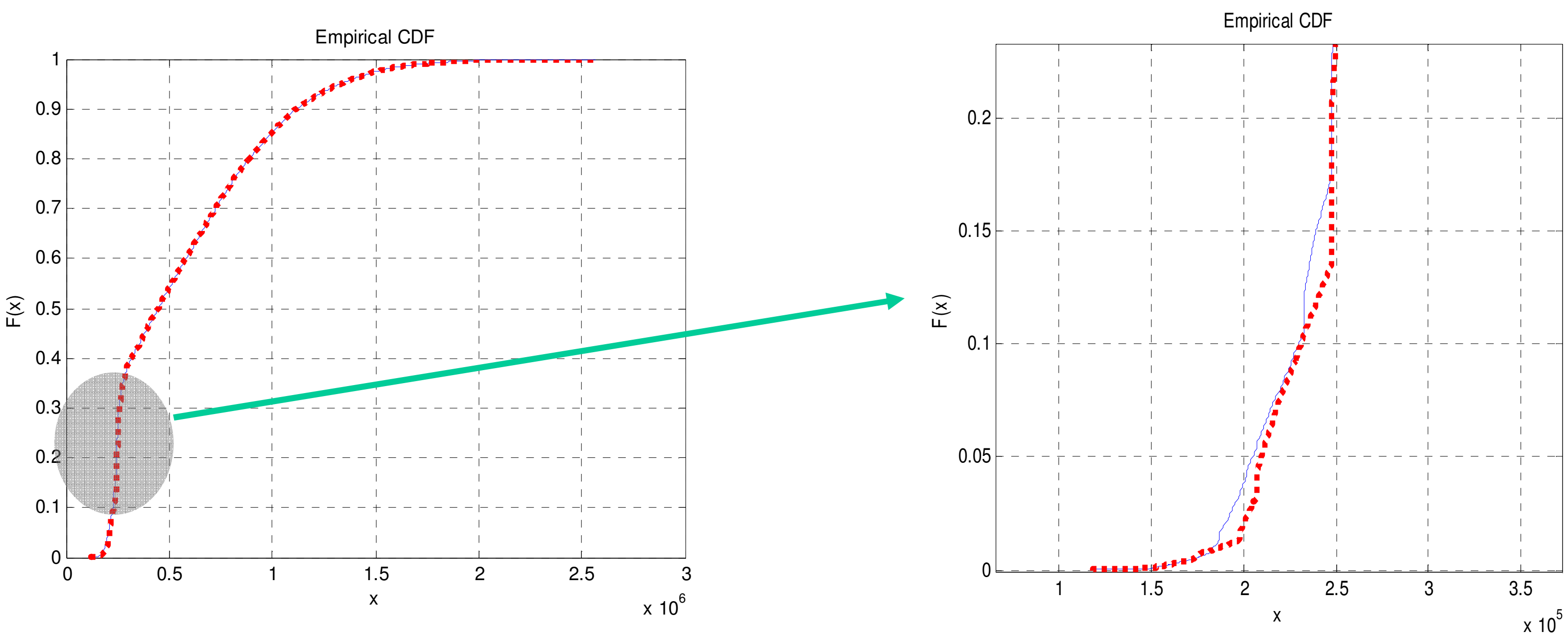
Case	Structure of the Tree	Scenario Distance Matrix (M)	Generated Power (MW)			
			Thermal	Difference (%)	Hydro	Difference (%)
OT	1*4*4*4*4*4*4: 4096	-	1,275	-	5,725	-
RT2	1*2*2*2*4*4*4: 512	Plant Capacity	1,275	0.0	5,725	0.0



- **KS-Test = 0.0027**

First stage decisions

Case	Structure of the Tree	Scenario Distance Matrix (M)	Generated Power (MW)			
			Thermal	Difference (%)	Hydro	Difference (%)
OT	1*4*4*4*4*4*4: 4096	-	1,275	-	5,725	-
RT2	1*2*2*2*4*4*4: 512	Plant Capacity	1,275	0.0	5,725	0.0



Conclusions & Next Steps

- The results indicate that the Nested Distance (quadratic process distance) can reduce the computational burden without compromising the expected operational cost and policy
- Improvements can be obtained when using others scenario distance matrices
- Apply the methodology in the context of the Brazilian Short-term Hydrothermal Scheduling problem
  - **Bigger system**
  - **The inflows are represented by an auto-regressive model (i.e., time-dependent)**

- R. M. Kovacevic, A. Pichler. Tree approximation for discrete time stochastic processes: a process distance approach. Annals of Operations Research, Vol. 235(1), 395-421, 2015
- G. Ch. Pflug, A. Pichler. A distance for multistage stochastic optimization models SIAM Journal on Optimization, 22, 1-23, 2012
- G. Ch. Pflug. Scenario tree generation for multiperiod financial optimization by optimal discretization. Mathematical Programming, 89:251–271, 2001
- Georg Ch. Pflug. Version-independence and nested distribution in multistage stochastic optimization. SIAM Journal on Optimization, 20:1406–1420, 2009
- J. Dupacová, N. Growe-Kuska, and W. Romisch. Scenario reduction in stochastic programming: An approach using probability metrics. Mathematical Programming, 95:493–511, 2003.
- H. Heitsch and W. Romisch. Scenario tree modeling for multistage stochastic programs. Mathematical Programming, 18:371–406, 2009.
- W. de Oliveira, C. Sagastizábal, et al. “Optimal scenario tree reduction for stochastic streamflows in power generation planning problems”. Optimization Methods & Software. Volume 25 Issue 6, December 2010.

Thank you!

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Work	Contributions	Probability metric
Dupacova et al (2003), Growe-Kuska et al. (2003); Heitsch e Römish (2005)	-Stability in two-stage stochastic programs -Heuristic-based algorithms for scenario reduction	Wasserstein distance
Heitsch e Römish (2009)	-Stability in multistage stochastic programs -Heuristic-based algorithm for scenario reduction	Wasserstein distance + Filtration distance
Oliveira; Sagastizábal, et al (2010)	-Combines sampling with scenario reduction -Stability in multistage stochastic programs	Wasserstein distance
Pflug (2009)	-Introduction of the nested distance	Nested distance
Pflug and Pichler (2012)	-Stability in multistage stochastic programs	Nested distance
Kovacevic and Pichler (2015)	-Stability in multistage stochastic programs -Heuristic-based algorithms for scenario reduction	Nested/Quadratic distance