LAMPS LABORATORY OF APPLIED MATHEMATICAL PROGRAMMING AND STATISTICS

PUC RIO

On the cost and side effects of time inconsistency in long-term hydrothermal planning

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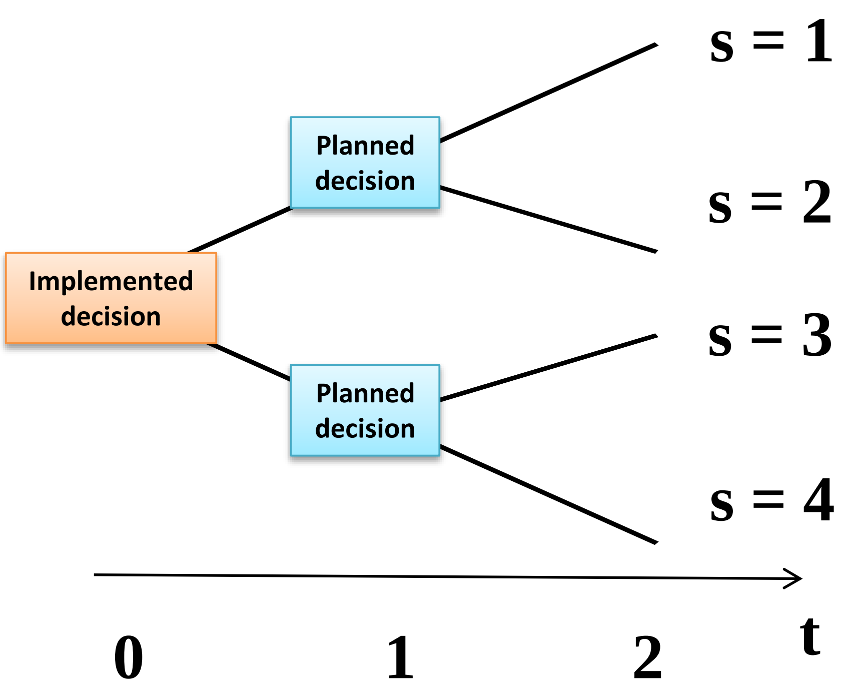
Arthur de Castro Brigatto
Alexandre Street
Davi M. Valladão

Time consistent policy

“a policy is time consistent if and only if future planned decisions are actually going to be implemented”



Time consistent policy

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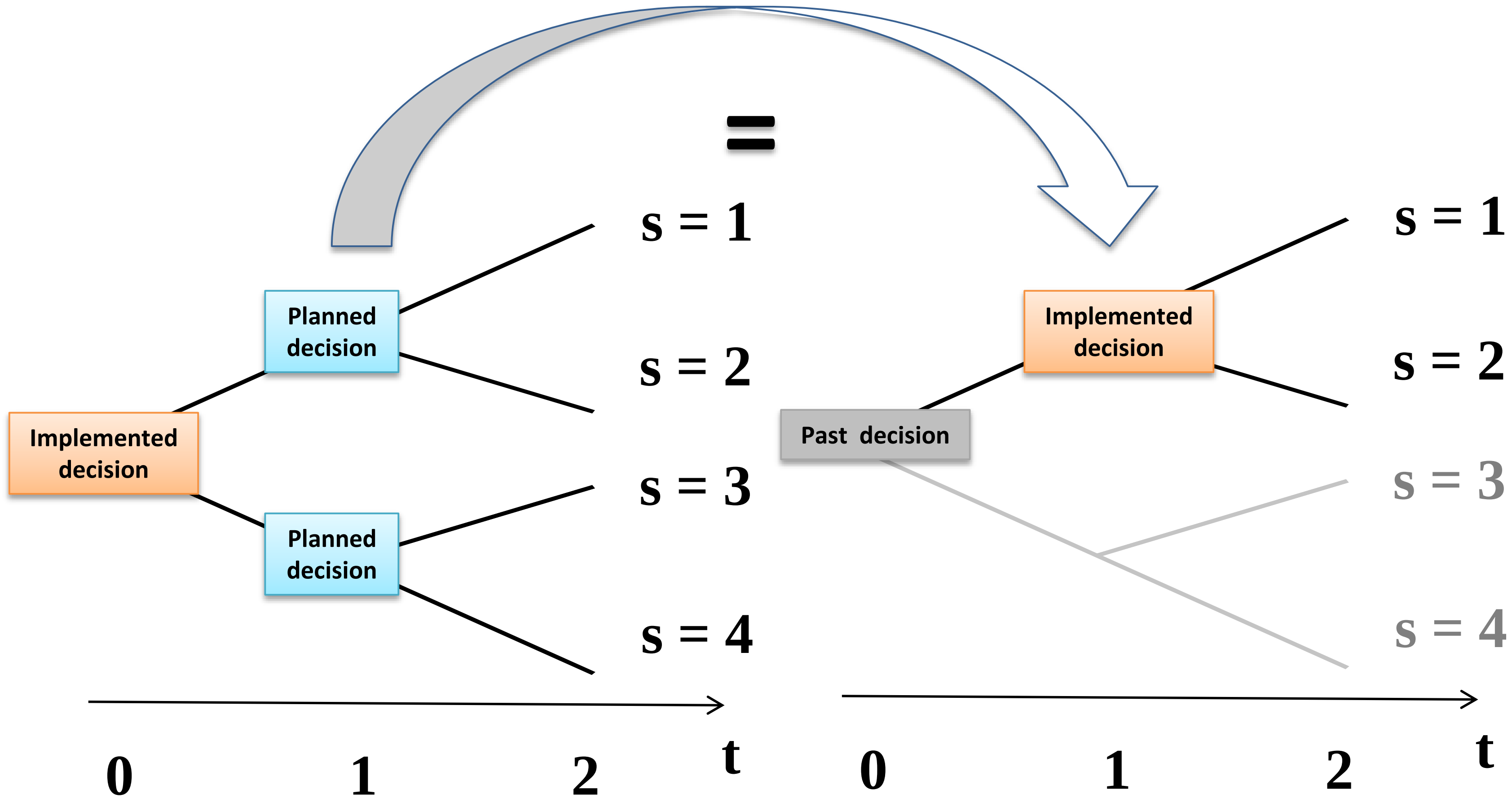
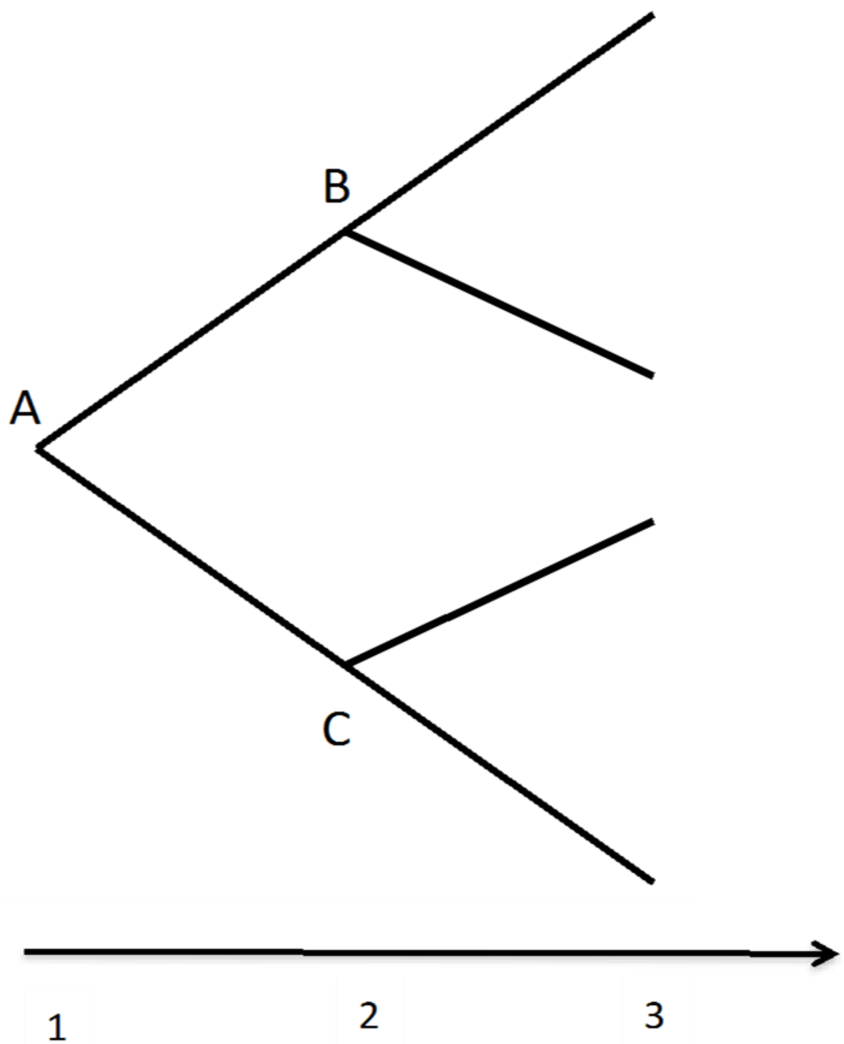
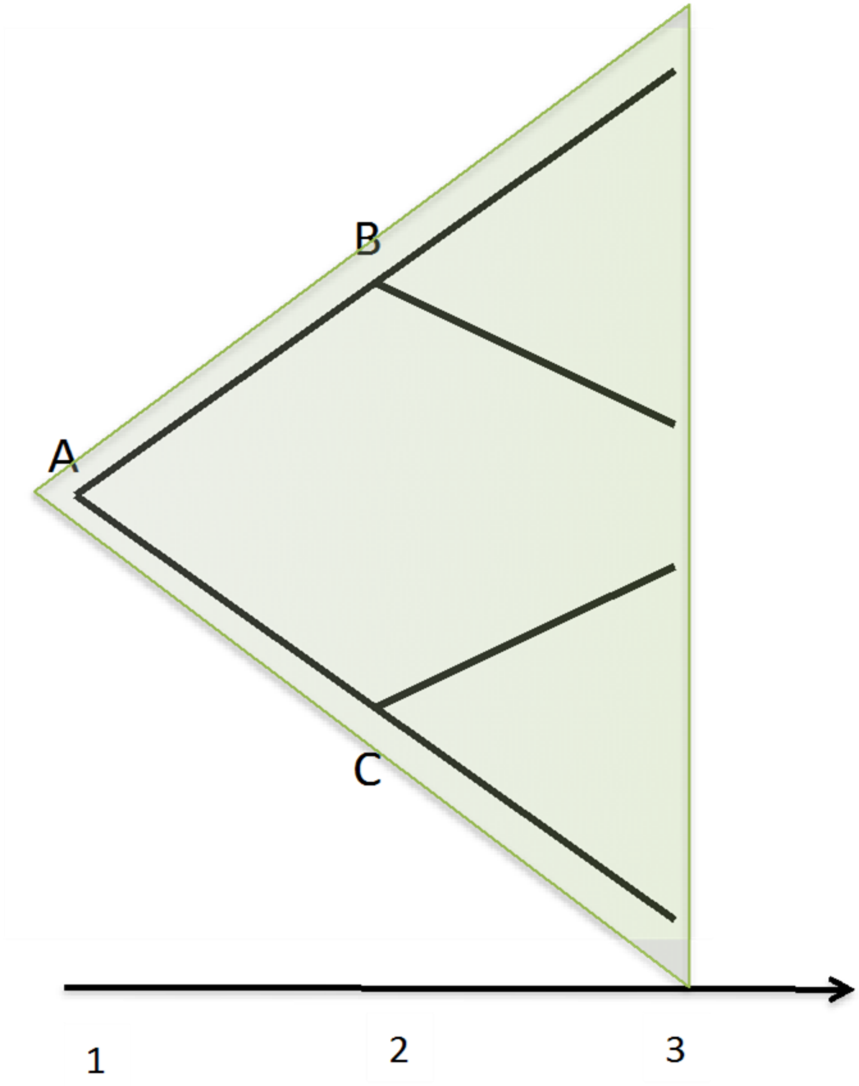


Dynamic stochastic programming context



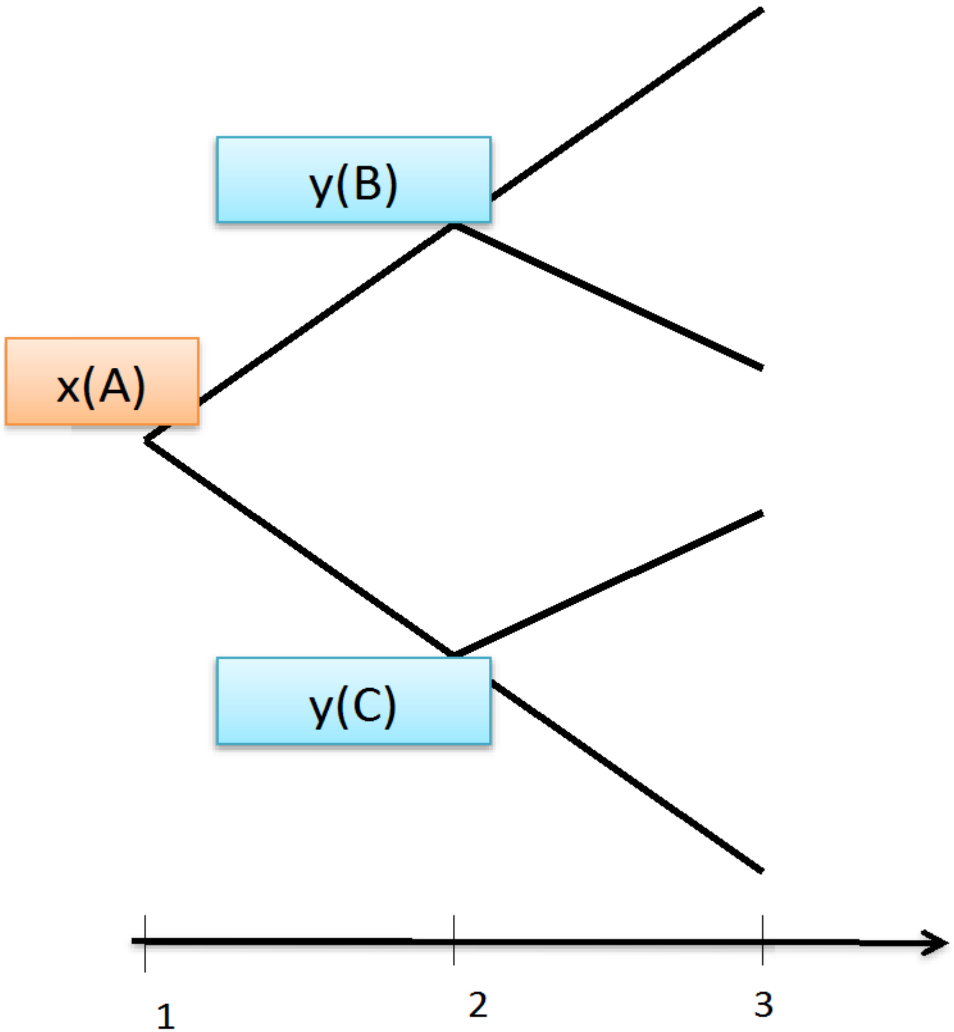
- For each node (A,B,C), decision maker:
  - Defines a multistage problem to be solved
    - P_1, P_2(B) and P_2(C)
  - Obtain
    - Implemented decision
    - Planned policy

Dynamic stochastic programming context



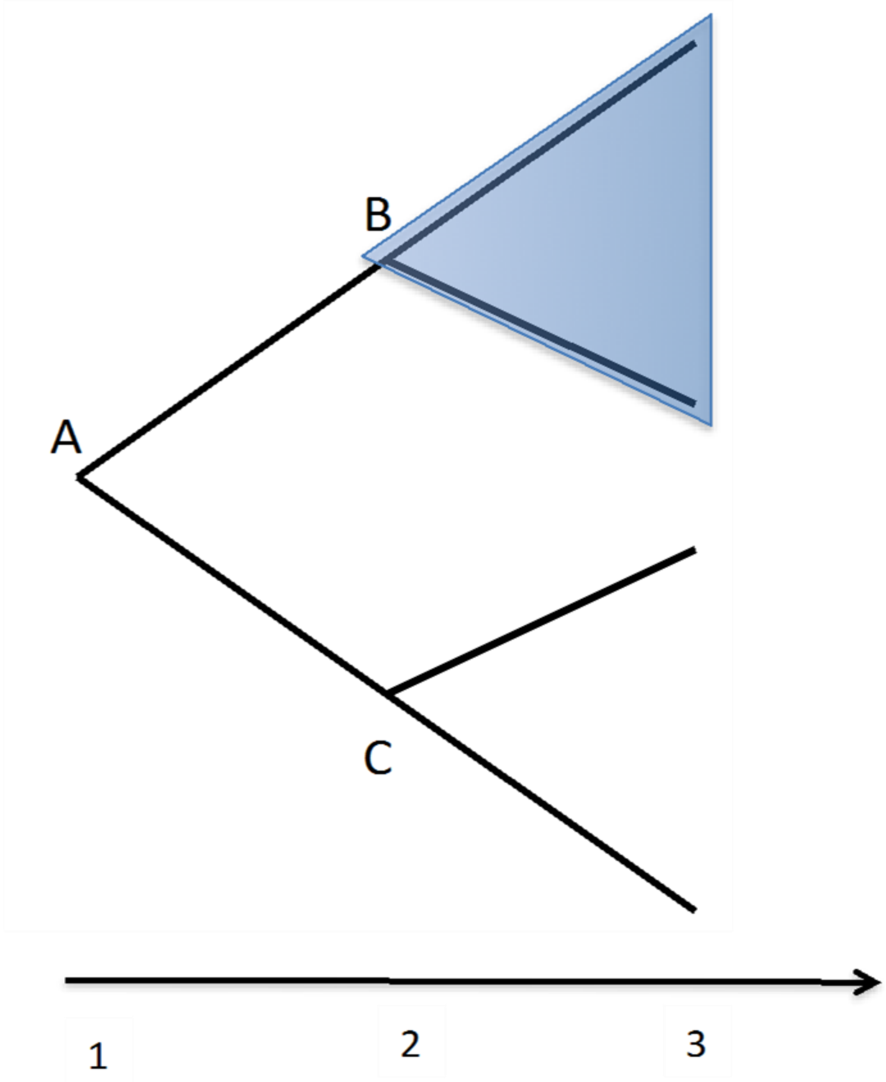
- For problem P_1, decision maker obtain:

Dynamic stochastic programming context



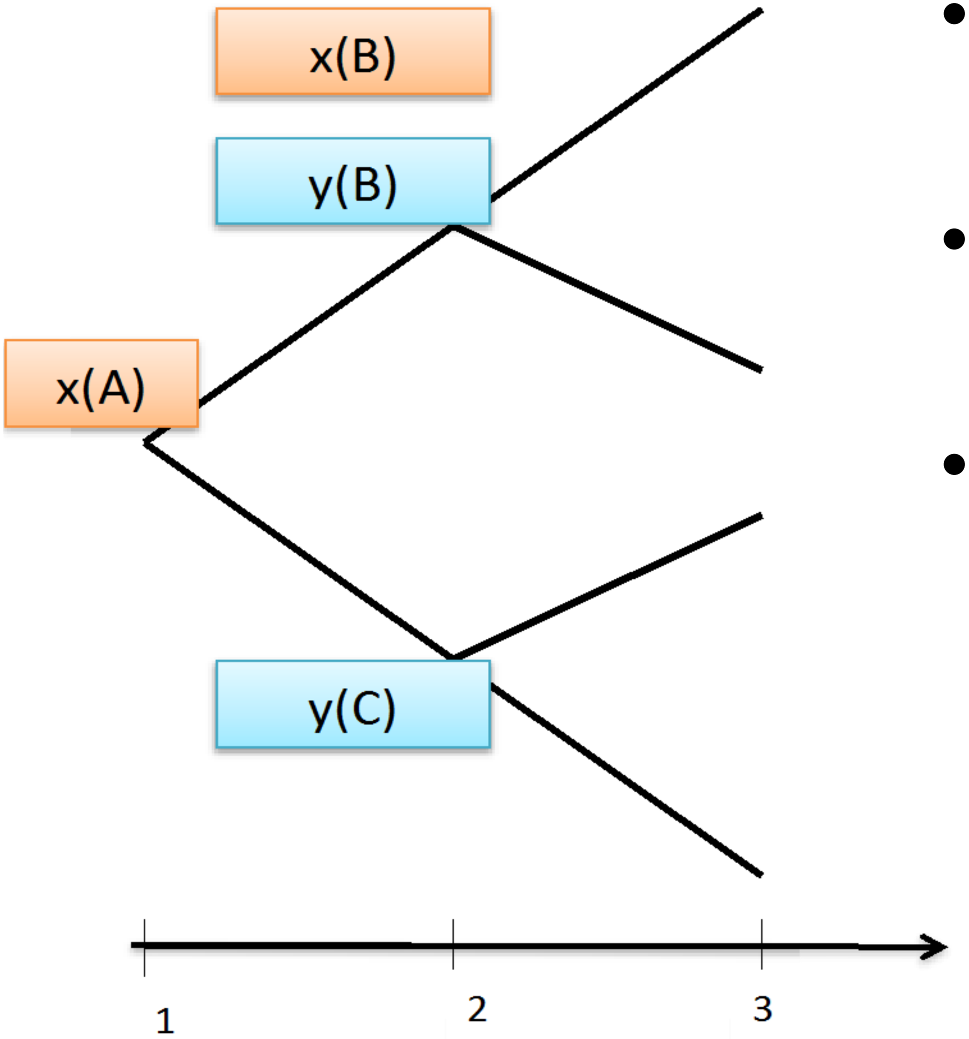
- For problem P_1, decision maker obtain:
  - Optimal solution of all nodes (A,B and C)
    - Implemented decision
      - **x(A)**
    - Planned decisions
      - y(B) and y(C)

Dynamic stochastic programming context



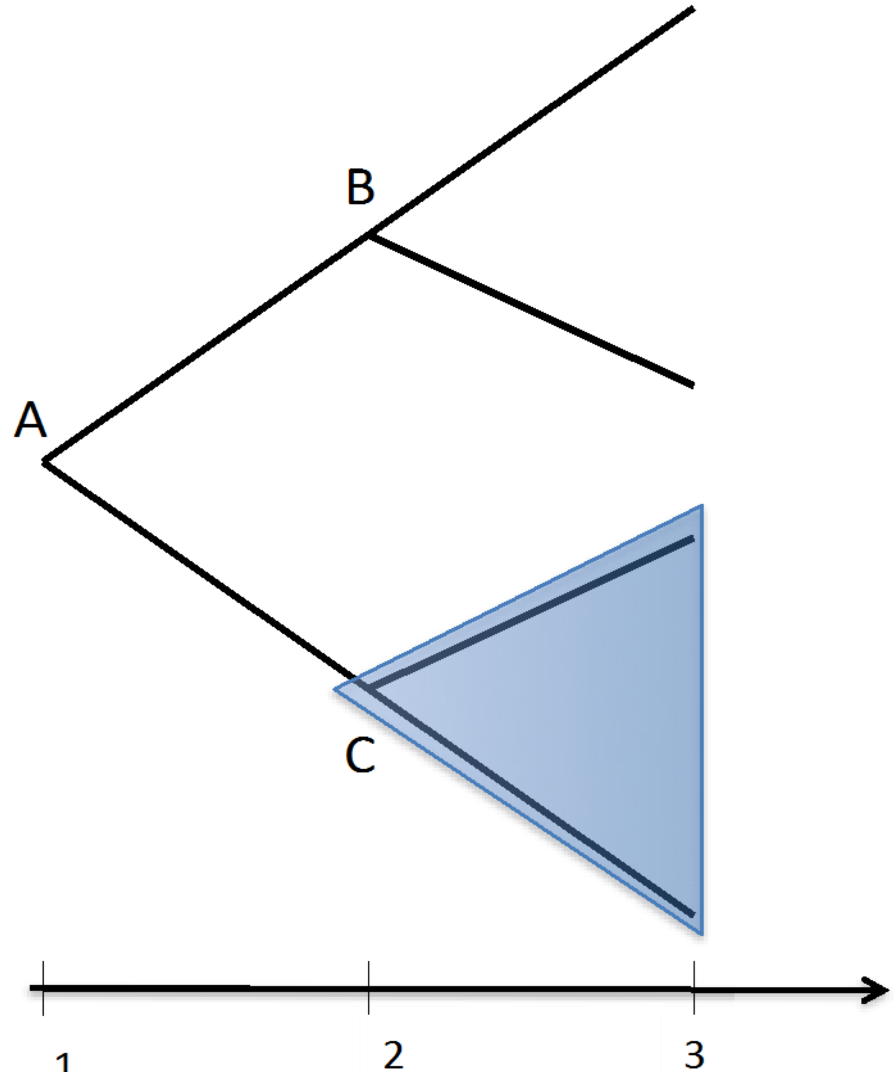
- Given the implemented decision x(A).
- Given uncertainty realization B
- Decision maker solves $P_2(B)$ and obtains:

Dynamic stochastic programming context



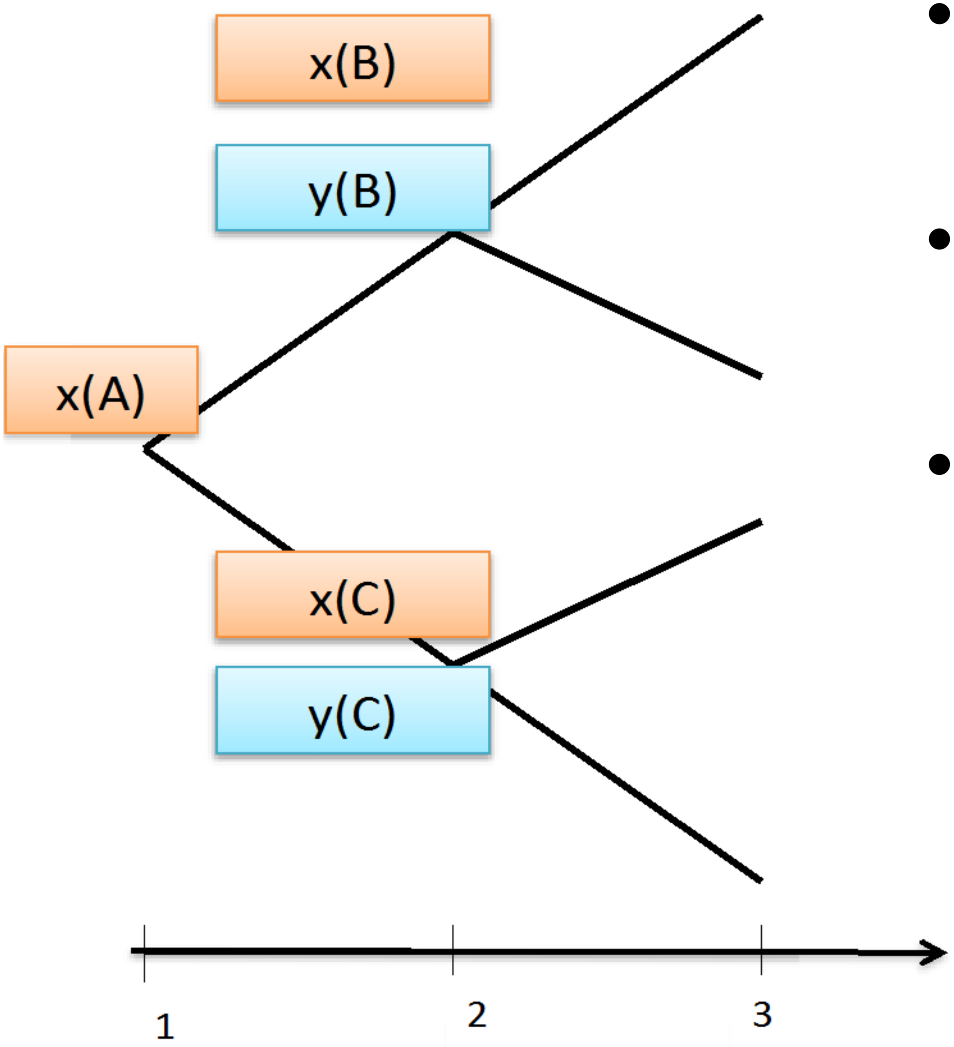
- Given the implemented decision x(A):
- Given uncertainty realization B
- Decision maker solves $P_2(B)$ and obtains:
  - The implemented decision
    - **x(B)**

Dynamic stochastic programming context



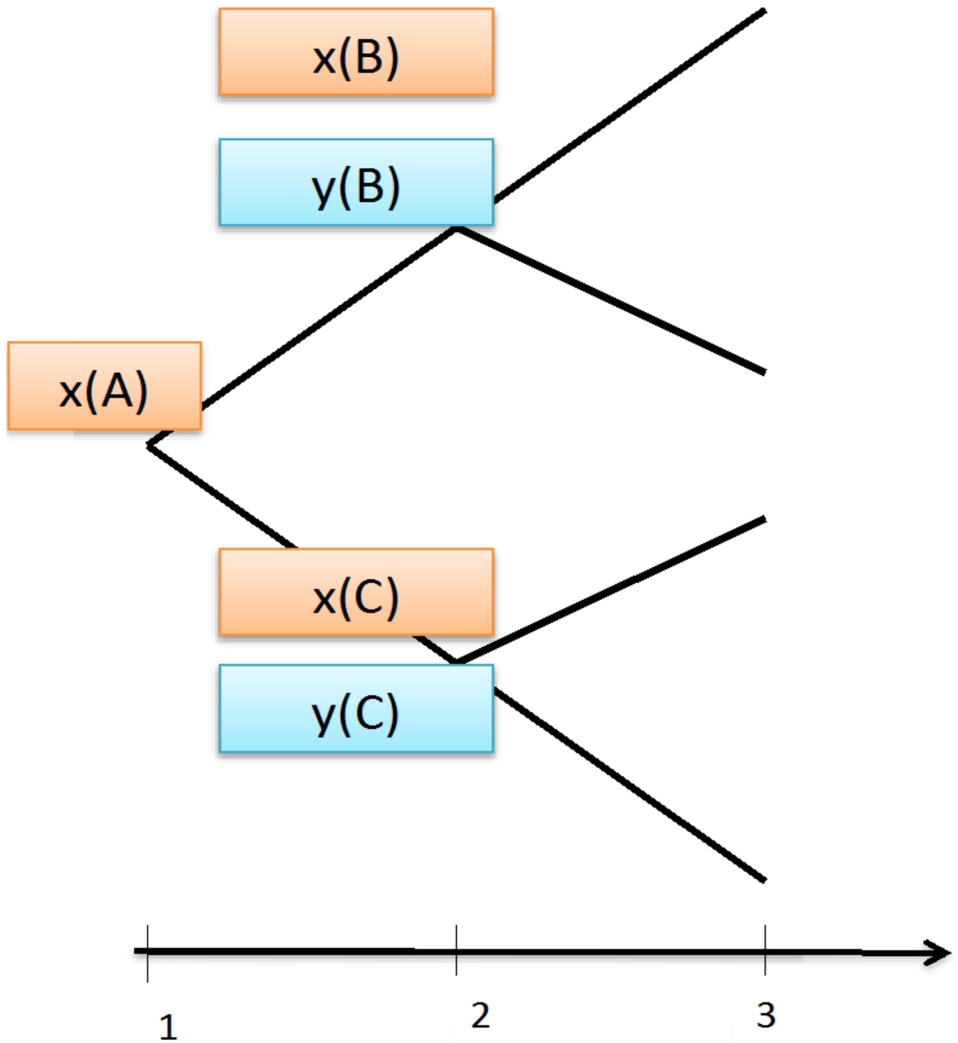
- Given the implemented decision x(A).
- Given uncertainty realization C
- Decision maker solves P_2(C) and obtains:

Dynamic stochastic programming context



- Given the implemented decision x(A).
- Given uncertainty realization C
- Decision maker solves $P_2(C)$ and obtains:
  - The implemented decision
    - **x(C)**

Dynamic stochastic programming context



- Time consistency requires

x(B) = y(B)

x(C) = y(C)

Objectives

To characterize modeling simplifications in the planning step, in contrast to the model used in the implementation step, as a possible source of time inconsistency

Stochastic Dual Dynamic Programming

- Uncertanties:
  - → Inflows: $\Omega_t = \{1,2,\ldots,N_t\}$ each with probability $p_{t,\omega}$.
- Sampling one scenario $\omega_t \in \Omega_t$:

$$Q_t(v_{t-1}, w_{t,\omega}) = \min_{g_t, y_t, f_t} c_t g_t + \mathcal{Q}_{t+1}(v_t)$$

Subject to:

$$A_t g_t + B_t y_t + C_t f_t = d_t$$
$$v_t + u_t + s_t + M(u_t + s_t) = v_{t-1} + w_{t,\omega} : (\pi_{t,\omega})$$
$$(y_t, g_t, f_t) \in \mathcal{X}_t \ .$$

→ $y_t = \begin{bmatrix} v_t \\ u_t \\ s_t \end{bmatrix}$

- $\mathcal{Q}_{t+1}(v_t) = \sum_{\omega \in \Omega_{t+1}} p_{t+1,\omega} \, Q_{t+1}(v_t, w_{t+1,\omega})$.

Forward step

$$\min_{g_t, y_t, f_t, \alpha_{t+1}} \; c_t g_t + \alpha_{t+1}$$

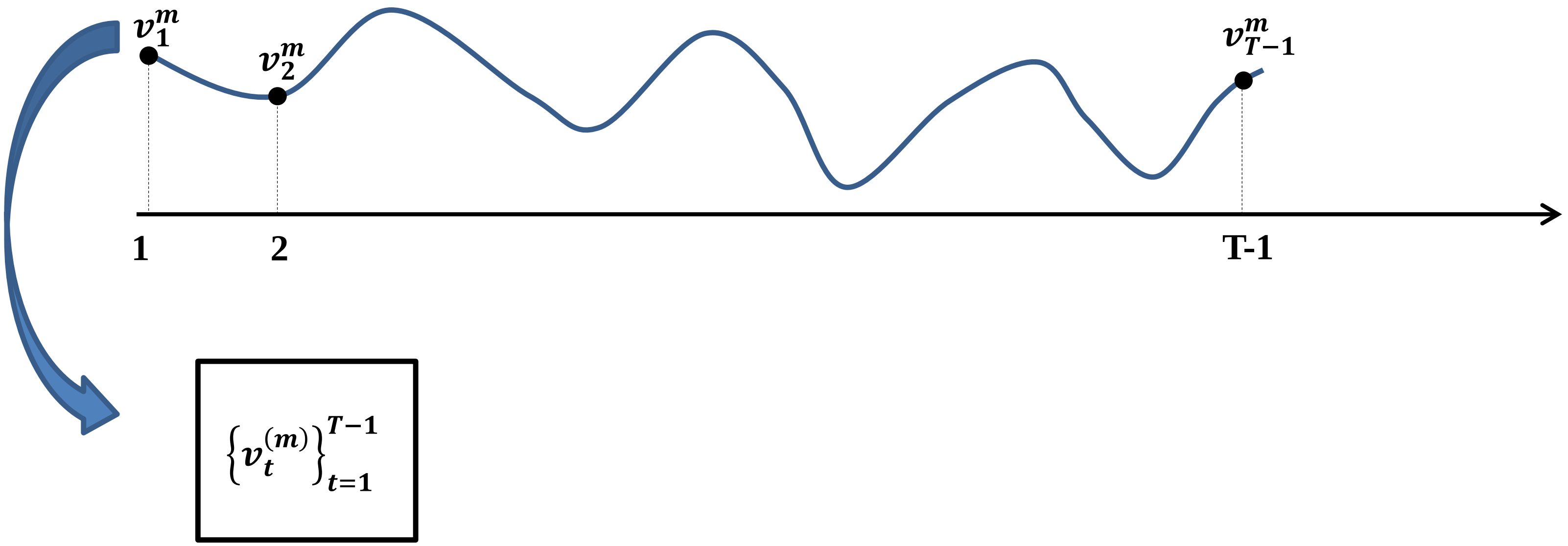
Subject to

$$A_t g_t + B_t y_t + C_t f_t = d_t$$

$$v_t + u_t + s_t + M(u_t + s_t) = v_{t-1}^{(m)} + w_{t,\omega}$$

$$\alpha_{t+1} \geq \tilde{Q}_{t+1}^{(k)}\left(v_t^{(k)}\right) + \left(\tilde{\pi}_{t+1}^{(k)}\right)^{\top}\left(v_t - v_t^{(k)}\right); \forall k \in \mathcal{K}^{(m)}$$

$$(y_t, g_t, f_t) \in \mathcal{X}_t \; .$$



Backward Step

$$\tilde{Q}_t^{(m)}\left(v_{(t-1)}, w_{t,\omega}\right) = \min_{g_t, y_t, f_t, \alpha_{t+1}} c_t g_t + \alpha_{t+1}$$

Subject to:

$$A_t g_t + B_t y_t + C_t f_t = d_t$$

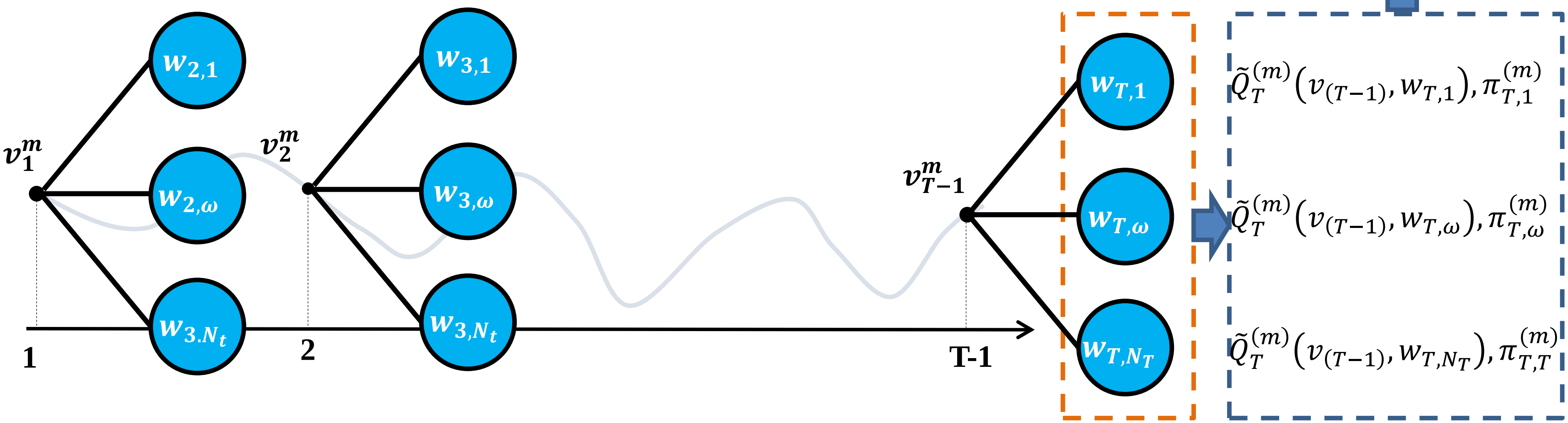
$$v_t + u_t + s_t + M(u_t + s_t) = v_{t-1}^{(m)} + w_{t,\omega} : (\tilde{\pi}_{t,\omega}^{(k)})$$

$$\alpha_{t+1} \geq \tilde{Q}_{t+1}^{(k)}\left(v_t^{(k)}\right) + \left(\tilde{\pi}_{t+1}^{(k)}\right)^{\top}\left(v_t - v_t^{(k)}\right); \forall k \in \mathcal{K}^{(m)}$$

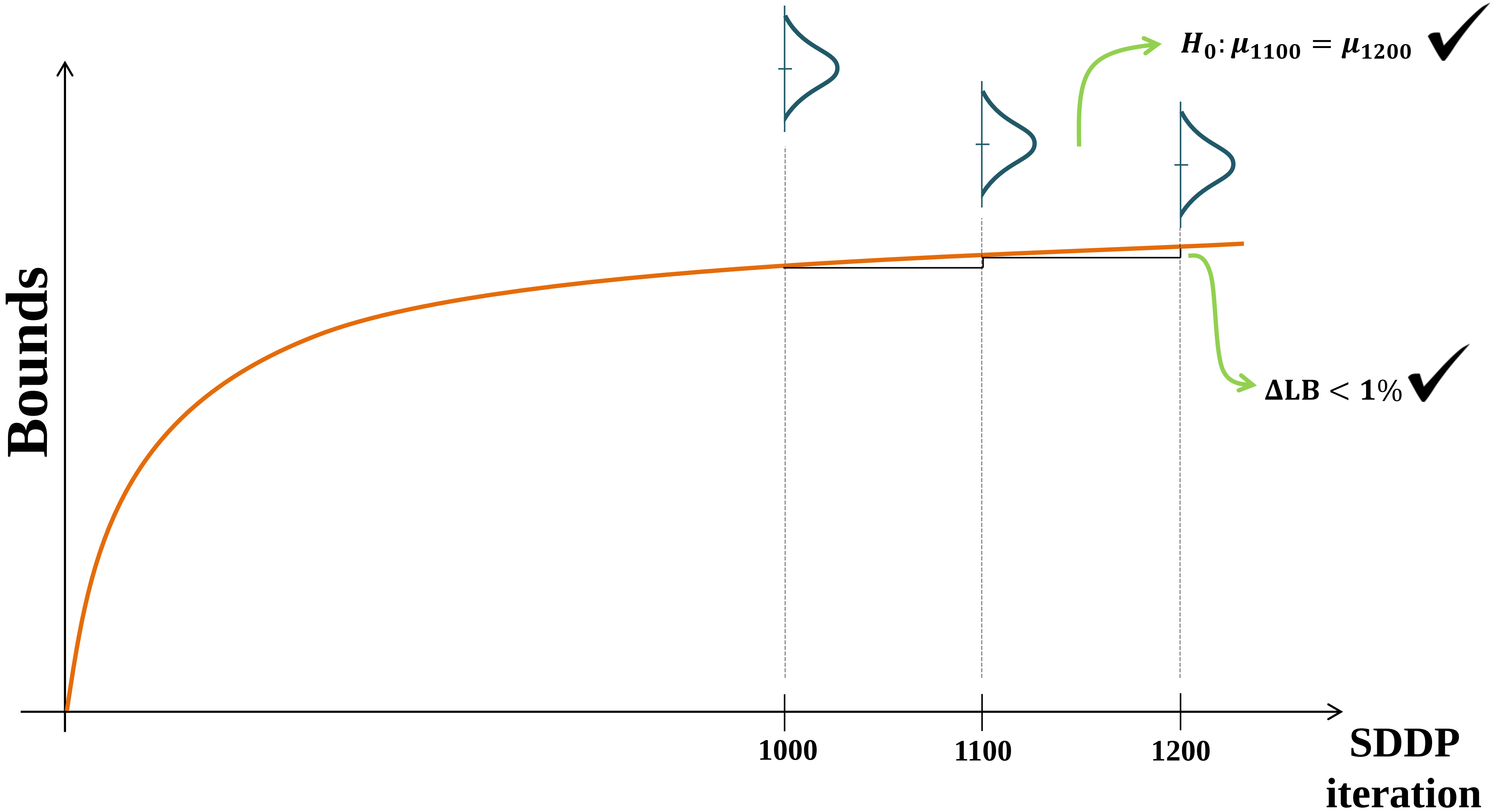
$$(y_t, g_t, f_t) \in \mathcal{X}_t \; .$$

$$\tilde{Q}_{t+1}^{(m)}\left(v_t^{(m)}\right) = \sum_{\omega \in \Omega_t} p_{t,\omega} \tilde{Q}_t^{(m)}\left(v_{(t-1)}, w_{t,\omega}\right)$$

$$\tilde{\pi}_{t+1}^{(m)} = \sum_{\omega \in \Omega_t} p_{t,\omega} \tilde{\pi}_{t,\omega}^{(k)}$$



Stopping Criterion



Framework

Implementation Model

$$\min_{g_t, y_t, f_t} c_t g_t + \mathcal{Q}_{t+1}^{plan}(v_t)$$

Subject to

$$A_t g_t + B_t y_t + C_t f_t = d_t$$

$$v_t + u_t + s_t + M(u_t + s_t)$$
$$= v_{t-1} + w_{t,\omega}; (\pi_{t,\omega})$$

$$(y_t, g_t, f_t) \in \mathcal{X}_t^{imp}.$$

Planning Model

$$\min_{g_t, y_t, f_t} c_t g_t + \mathcal{Q}_{t+1}^{plan}(v_t)$$

Subject to

$$A_t g_t + B_t y_t + C_t f_t = d_t$$

$$v_t + u_t + s_t + M(u_t + s_t)$$
$$= v_{t-1} + w_{t,\omega}; (\pi_{t,\omega})$$

$$(y_t, g_t, f_t) \in \mathcal{X}_t^{plan}.$$

Planning Step in t

- Solve SDDP using a planning model (simplified)

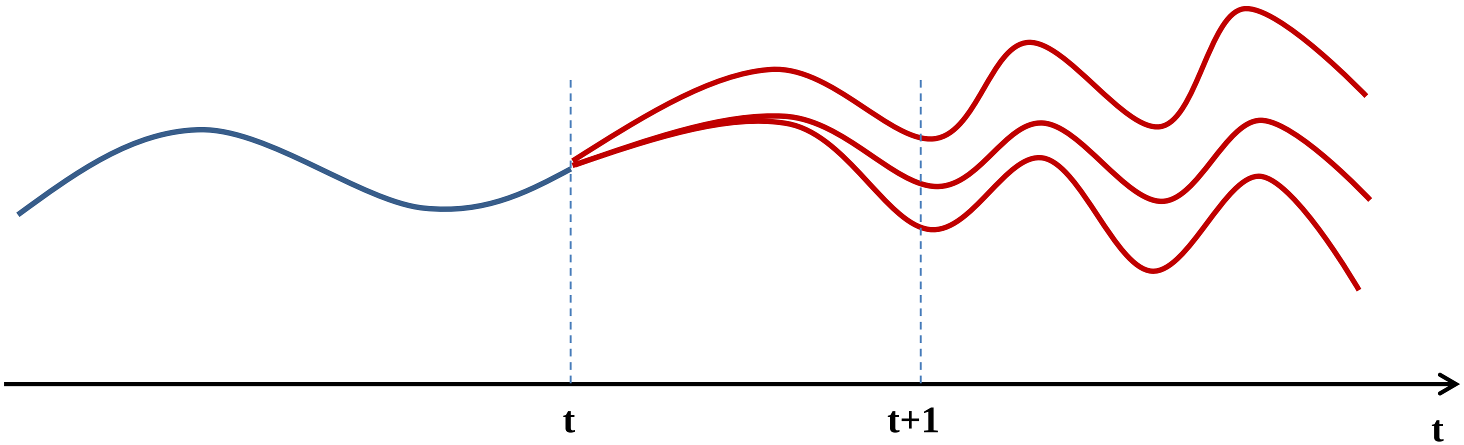
$$\min_{g_t, y_t, f_t} c_t g_t + \mathcal{Q}_{t+1}^S(v_t)$$

Subject to:

$$A_t g_t + B_t y_t + C_t f_t = d_t$$

$$v_t + u_t + s_t + M(u_t + s_t) = v_{t-1} + w_{t,\omega}; (\pi_{t,\omega})$$

$$(y_t, g_t, f_t) \in \mathcal{X}_t^S.$$



Planning Step in t

- Obtain the recourse function for $t + 1$

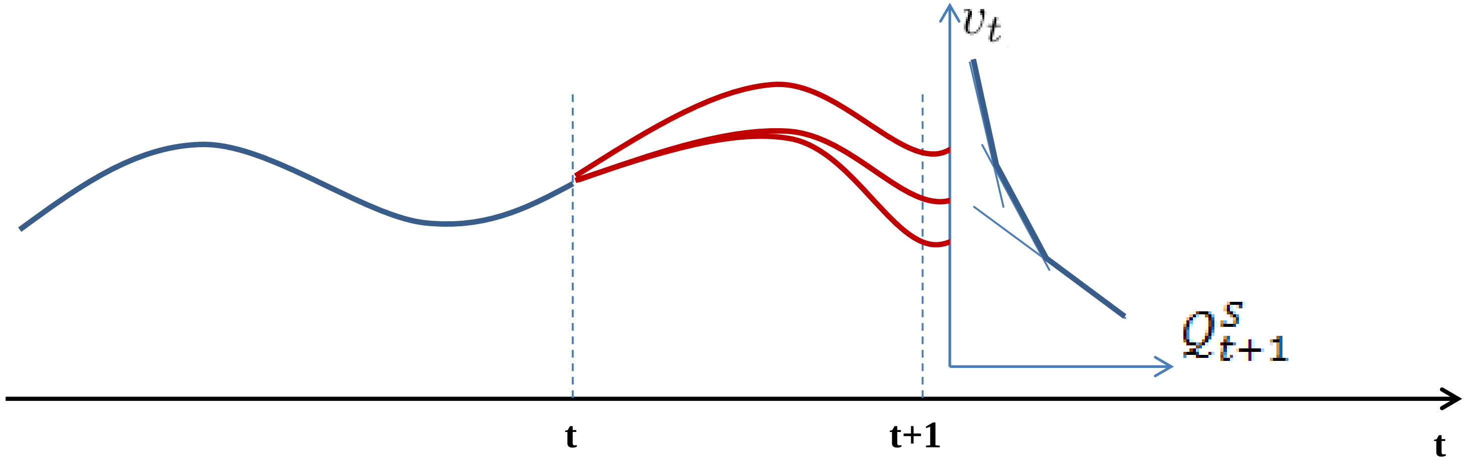
$$\min_{g_t, y_t, f_t} c_t g_t + \mathcal{Q}_{t+1}^S(v_t)$$

Subject to

$$A_t g_t + B_t y_t + C_t f_t = d_t$$

$$v_t + u_t + s_t + M(u_t + s_t) = v_{t-1} + w_{t,\omega}; (\pi_{t,\omega})$$

$$(y_t, g_t, f_t) \in \mathcal{X}_t^S.$$



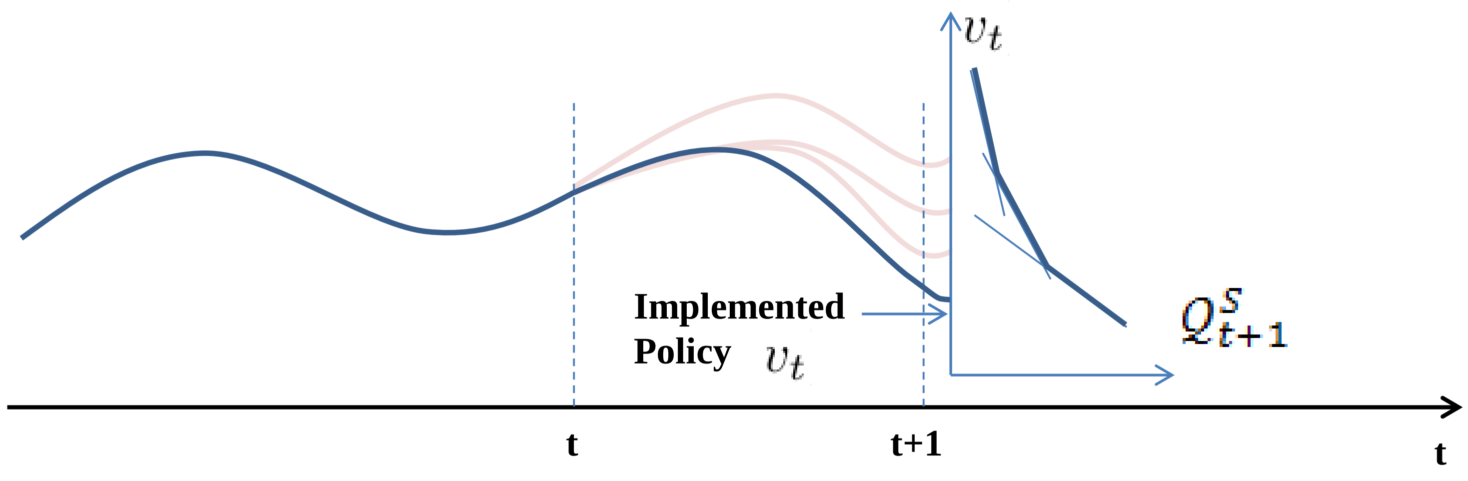
Implementation step in t

- Uses the recourse function (simplified) for $t+1$ and implements the first stage decision using a detailed model.

$$\min_{g_t, y_t, f_t} c_t g_t + \mathcal{Q}^S_{t+1}(v_t)$$

Subject to:

$$A_t g_t + B_t y_t + C_t f_t = d_t$$
$$v_t + u_t + s_t + M(u_t + s_t) = v_{t-1} + w_{t,\omega};\ (\pi_{t,\omega})$$
$$(y_t, g_t, f_t) \in \mathcal{X}^D_t.$$



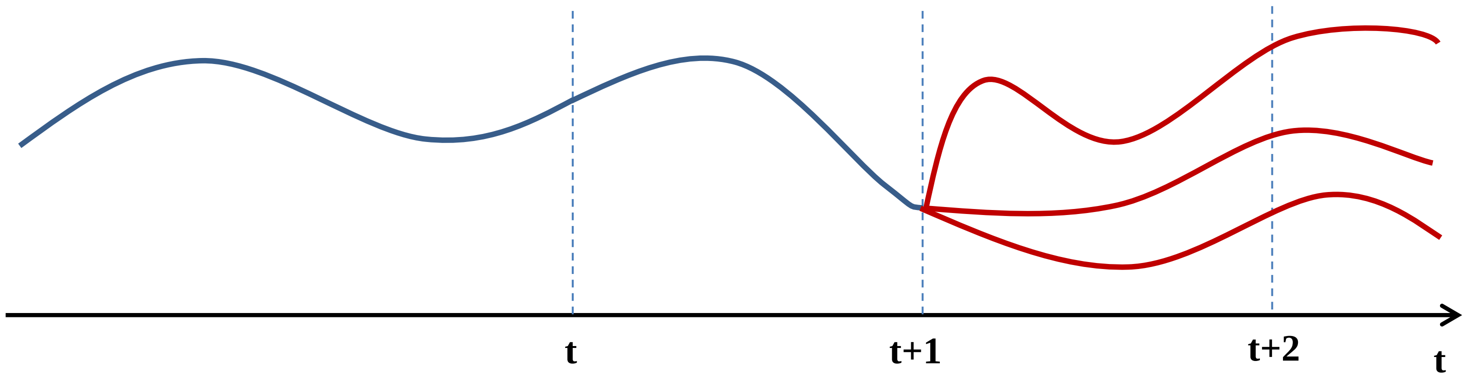
Planning Step in $t + 1$

- Solve SDDP using a planning model (simplified)

$$\min_{g_{t+1}, y_{t+1}, f_{t+1}} c_{t+1} g_t + \mathcal{Q}^S_{t+\mathbf{2}}(v_{t+1})$$

Subject to:

$$A_{t+1} g_{t+1} + B_{t+1} y_{t+1} + C_{t+1} f_{t+1} = d_{t+1}$$
$$v_t + u_t + s_t + M(u_t + s_t) = v_t + w_{t+1,\omega}; (\pi_{t+1,\omega})$$
$$(y_{t+1}, g_{t+1}, f_{t+1}) \in \mathcal{X}^S_{t+1}.$$



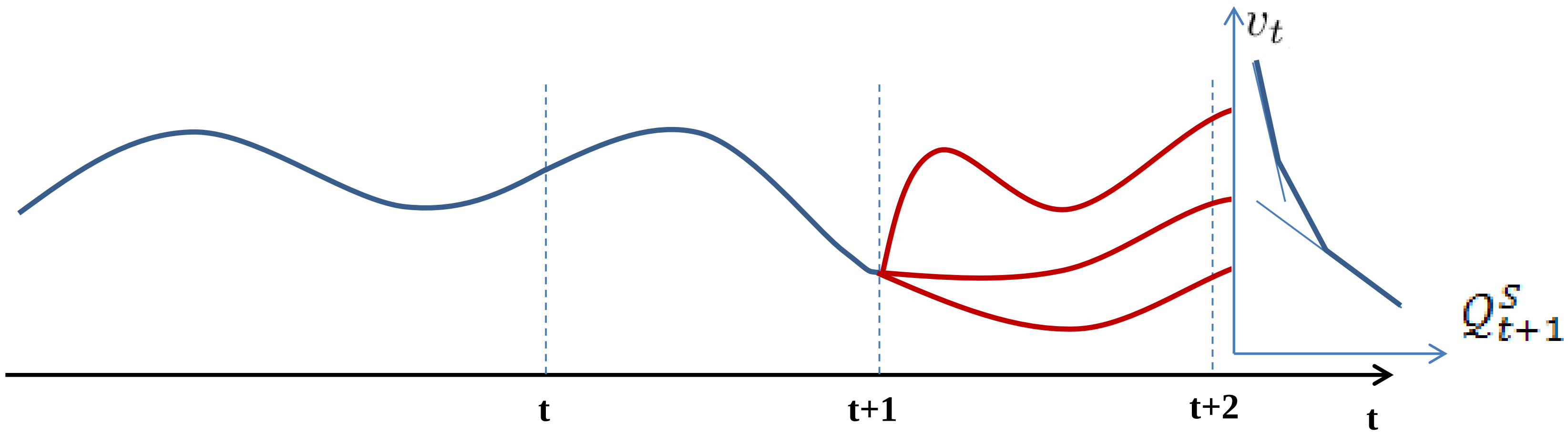
Planning Step in $t+1$

- Obtain the recourse function for $t+2$

$$\min_{g_{t+1},y_{t+1},f_{t+1}} c_{t+1}g_t + \mathcal{Q}^S_{t+\mathbf{2}}(v_{t+1})$$

Subject to:

$$A_{t+1}g_{t+1} + B_{t+1}y_{t+1} + C_{t+1}f_{t+1} = d_{t+1}$$
$$v_t + u_t + s_t + M(u_t + s_t) = v_t + w_{t+1,\omega};\ (\pi_{t+1,\omega})$$
$$(y_{t+1}, g_{t+1}, f_{t+1}) \in \mathcal{X}^S_{t+1}.$$



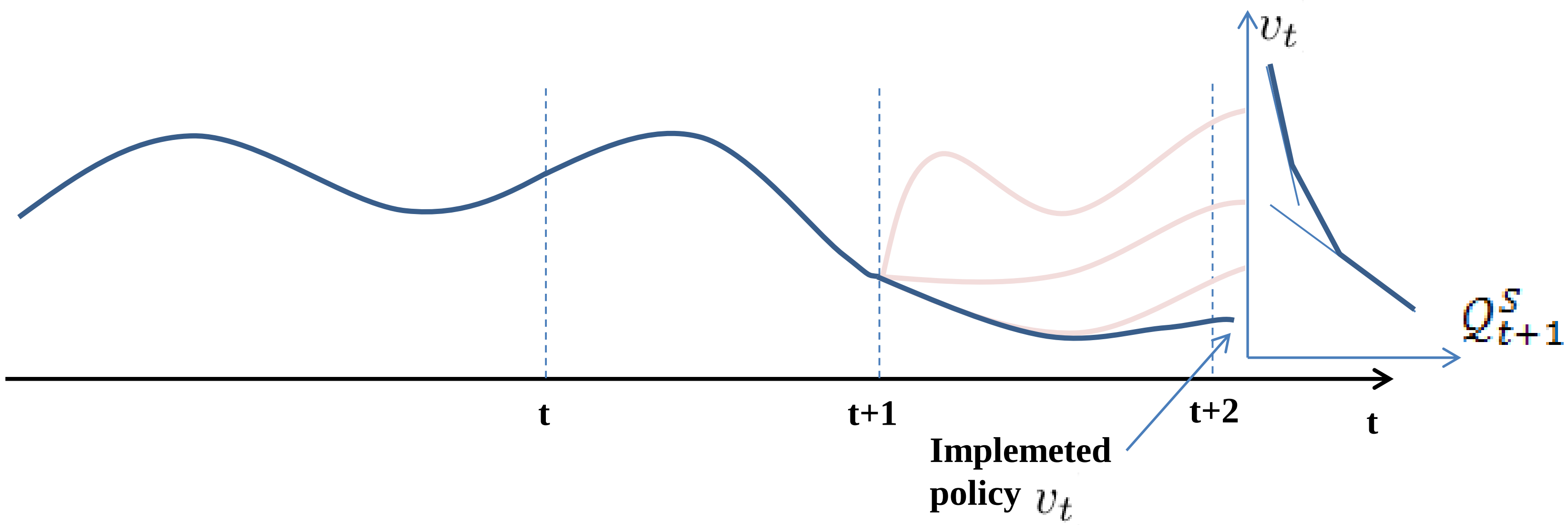
Implementation step in $t+1$

- Uses the recourse function (simplified) for $t+2$ and implements the first stage decision using a detailed model.

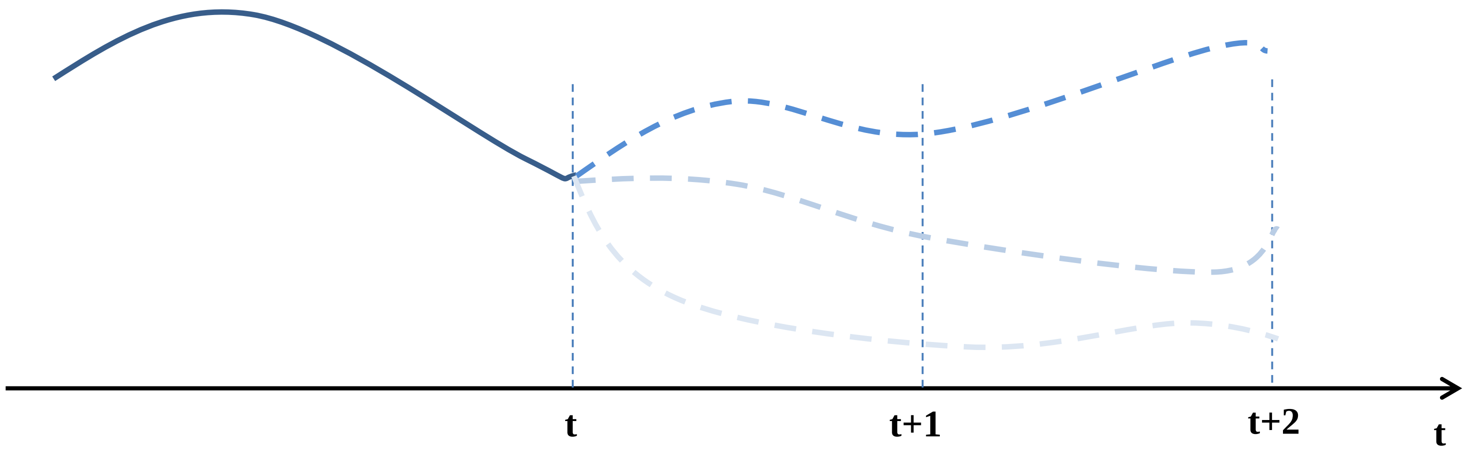
$$\min_{g_{t+1}, y_{t+1}, f_{t+1}} c_{t+1} g_t + \mathcal{Q}^S_{t+\mathbf{2}}(v_{t+1})$$

Subject to:

$$A_{t+1} g_{t+1} + B_{t+1} y_{t+1} + C_{t+1} f_{t+1} = d_{t+1}$$
$$v_t + u_t + s_t + M(u_t + s_t) = v_t + w_{t+1,\omega};\ (\pi_{t+1,\omega})$$
$$(y_{t+1}, g_{t+1}, f_{t+1}) \in \mathcal{X}^D_{t+1}.$$



Uncertainty in inflow realization in the implementation step



$$\mathcal{P}(\{Q_{t+1}^{plan}\}_{t=1}^{T}, \{\mathcal{X}_t^{imp}\}_{t=1}^{T}, \{w_{t,\omega}\}_{t,\omega=1}^{T,M}).$$

Simulating the implemented policy

SDDP method extended to a rolling-horizon scheme

1: Sample M inflow paths, $\{\mathrm{w}_{t,\omega}\}_{t,\omega=1}^{T,M}$.
2: Set $t = 1$ and initial conditions to $\{v_{0,\omega}\}_{\omega=1}^{M}$.
3: **for** each sampled path $\omega = 1, \ldots, M$ **do**
4:     Converge SDDP with $\mathcal{X}_\tau \leftarrow \mathcal{X}_\tau^{plan}\ \forall \tau \geq t$.
5:     Store the recourse function $\mathcal{Q}_{t+1}^{plan}$.
6:     Solve the implementation problem for period t using $\mathcal{Q}_{t+1}^{plan}$.
7:     Update $\mathcal{P}(\{\mathcal{Q}_t^{plan}\}_{t=1}^{T}, \{\mathcal{X}_t^{imp}\}_{t=1}^{T}, \{\mathrm{w}_{t,\omega}\}_{t,\omega=1}^{T,M})$.
8: **end for**
9: $t \leftarrow t + 1$.
10: **if** $t = T + 1$ **then**
11:     STOP.
12: **else**
13:     Set initial conditions to $\{v_{t-1,\omega}^*\}_{\omega=1}^{M}$ stored in $\mathcal{P}$.
14:     Go to step 3.
15: **end if**

Fast algorithm – Forward Step

$$\min_{g_t, y_t, f_t, \alpha_{t+1}} c_t g_t + \alpha_{t+1}$$

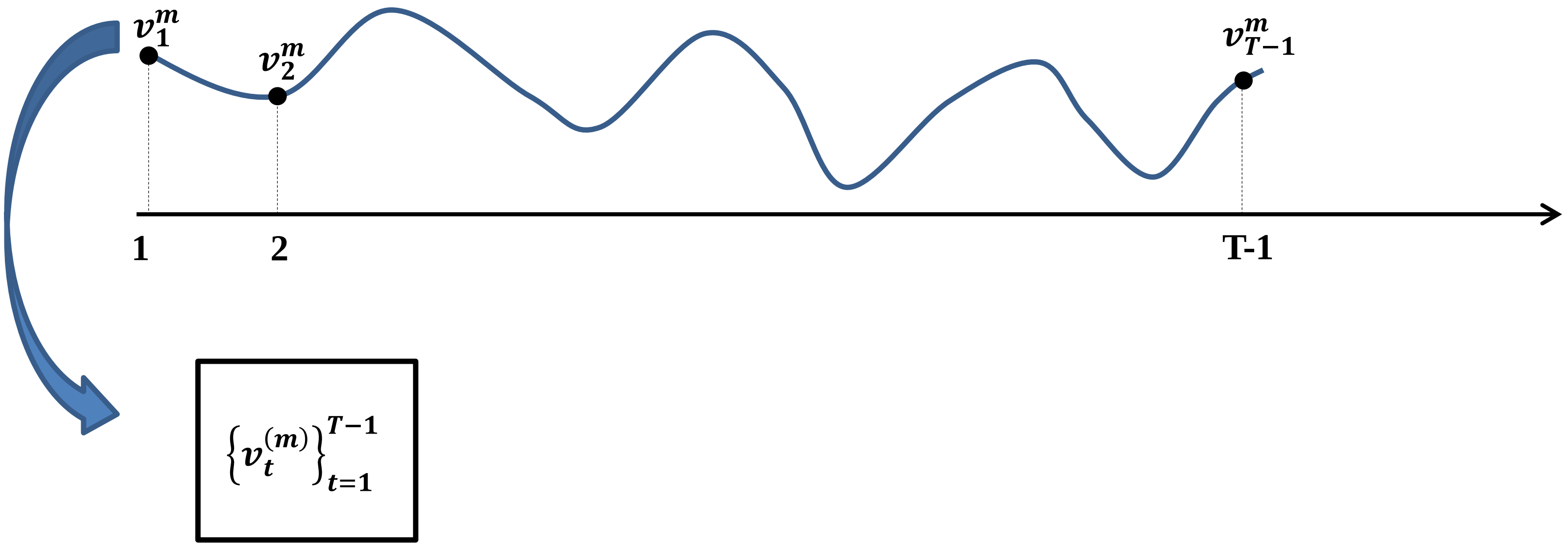
Subject to:

$$A_t g_t + B_t y_t + C_t f_t = d_t$$

$$v_t + u_t + s_t + M(u_t + s_t) = v_{t-1}^{(m)} + w_{t,\omega}$$

$$\alpha_{t+1} \geq \tilde{Q}_{t+1}^{(k)}\left(v_t^{(k)}\right) + \left(\tilde{\pi}_{t+1}^{(k)}\right)^{\top}\left(v_t - v_t^{(k)}\right); \forall k \in \mathcal{K}^{(m)}$$

$$(y_t, g_t, f_t) \in \mathcal{X}^D{}_t \; .$$



Fast Algorithm – Backward Step

$$\tilde{Q}_t^{(m)}\left(v_{(t-1)}, w_{t,\omega}\right) = \min_{g_t, y_t, f_t, \alpha_{t+1}} c_t g_t + \alpha_{t+1}$$

sujeito a:

$$A_t g_t + B_t y_t + C_t f_t = d_t$$

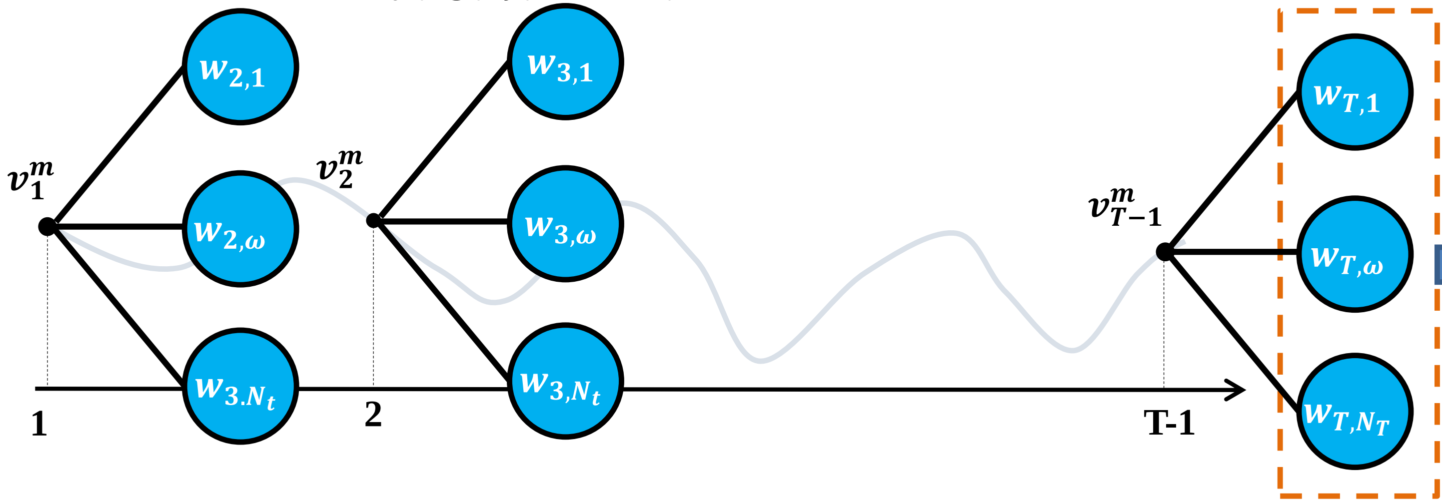
$$v_t + u_t + s_t + M(u_t + s_t) = v_{t-1}^{(m)} + w_{t,\omega} : (\tilde{\pi}_{t,\omega}^{(k)})$$

$$\alpha_{t+1} \geq \tilde{Q}_{t+1}^{(k)}\left(v_t^{(k)}\right) + \left(\tilde{\pi}_{t+1}^{(k)}\right)^{\mathsf{T}}\left(v_t - v_t^{(k)}\right); \forall k \in \mathcal{K}^{(m)}$$

$$(y_t, g_t, f_t) \in \mathcal{X}^S{}_t \; .$$

$$\tilde{Q}_{t+1}^{(m)}\left(v_t^{(m)}\right) = \sum_{\omega \in \Omega_t} p_{t,\omega} \tilde{Q}_t^{(m)}\left(v_{(t-1)}, w_{t,\omega}\right)$$

$$\tilde{\pi}_{t+1}^{(m)} = \sum_{\omega \in \Omega_t} p_{t,\omega} \tilde{\pi}_{t,\omega}^{(k)}$$



$$\tilde{Q}_T^{(m)}\left(v_{(T-1)}, w_{T,1}\right), \pi_{T,1}^{(m)}$$

$$\tilde{Q}_T^{(m)}\left(v_{(T-1)}, w_{T,\omega}\right), \pi_{T,\omega}^{(m)}$$

$$\tilde{Q}_T^{(m)}\left(v_{(T-1)}, w_{T,N_T}\right), \pi_{T,T}^{(m)}$$

Time Inconsistency Gap

- The inconsistency gap is:

$$GAP = \frac{1}{M}\sum_{t=1}^{T}\sum_{\omega=1}^{M} c_t^\top g_{t,\omega}^D - \frac{1}{M}\sum_{t=1}^{T}\sum_{\omega=1}^{M} c_t^\top g_{t,\omega}^S$$

- There exists the possibilitity that the gap is due to a sampling error:

$$\begin{cases} H_0: \mu^D = \mu^S \\ H_1: \mu^D \neq \mu^S \end{cases}$$

- The null hypothesis is accepted if $0 \in [GAP \pm 1.96 \cdot \sqrt{(\frac{S_D^2 + S_S^2}{M})}]$

Simplification Example – Kircchoff's Voltage Law

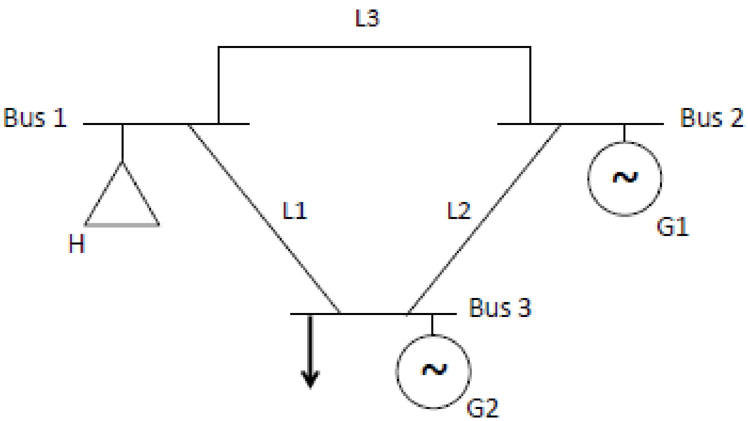
- Simplified model $\mathcal{X}_t^S$:

$$\mathcal{X}_t^S = \{ (y_t, g_t, f_t) | \\ \underline{V} \le v_t \le \overline{V} \\ \underline{U} \le u_t \le \overline{U} \\ \underline{S} \le s_t \le \overline{S} \\ \underline{G} \le g_t \le \overline{G} \\ -\overline{F} \le f_t \le \overline{F} \}.$$

- Detailed model $\mathcal{X}_t^D$:

$$\mathcal{X}_t^D = \mathcal{X}_t^S \cap \{(y_t, g_t, f_t, \theta_t) | f_t = S\theta_t\}.$$

System



- $T = 60$. (Last 12 periods are discarded).
- D = 100MWh.

Thermal Generator	c R$/MWh	$\overline{G}$ MW
G_1	20	100
G_2	100	55

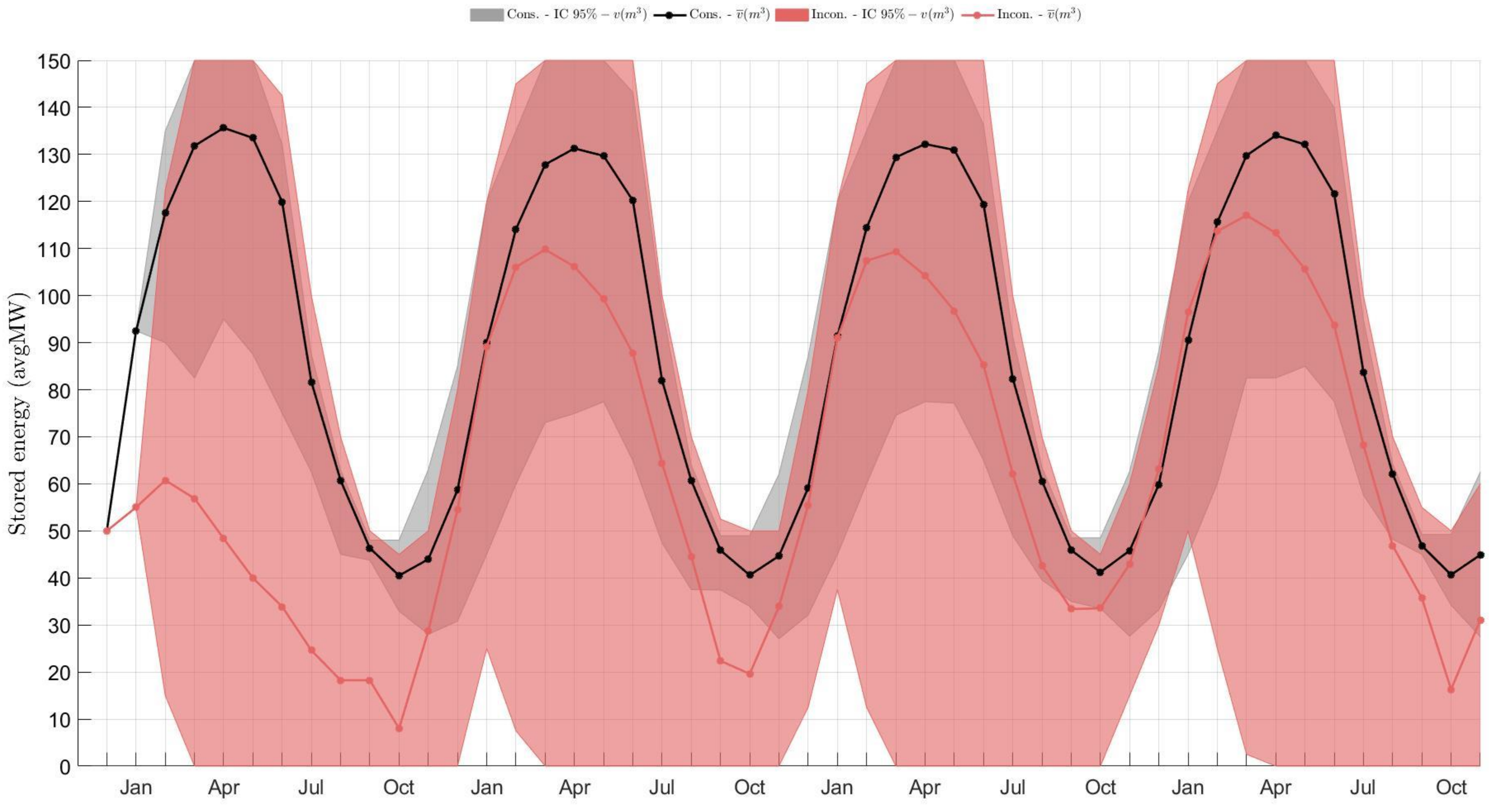
Hydro Generator	$\overline{V}$ m^3	$\overline{U}$ m^3
H	150	80

TL	From	To	$\overline{F}$ MW	Reactance (pu)
1	1	3	100	1
2	2	3	65	0.5
3	1	2	25	1

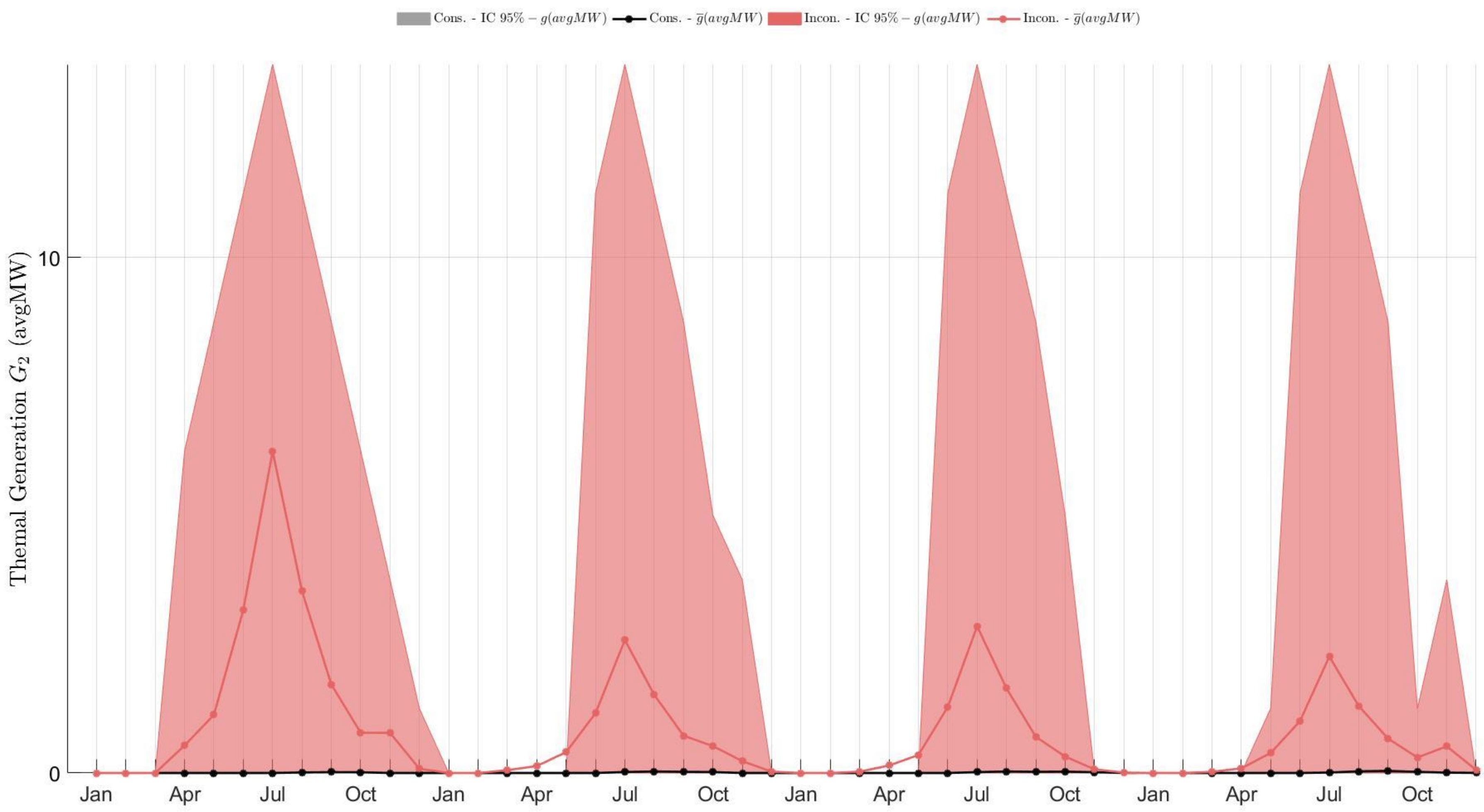
GAP

GAP: MMR$174.4
GAP confidence interval: MMR$[174.05 174.55]

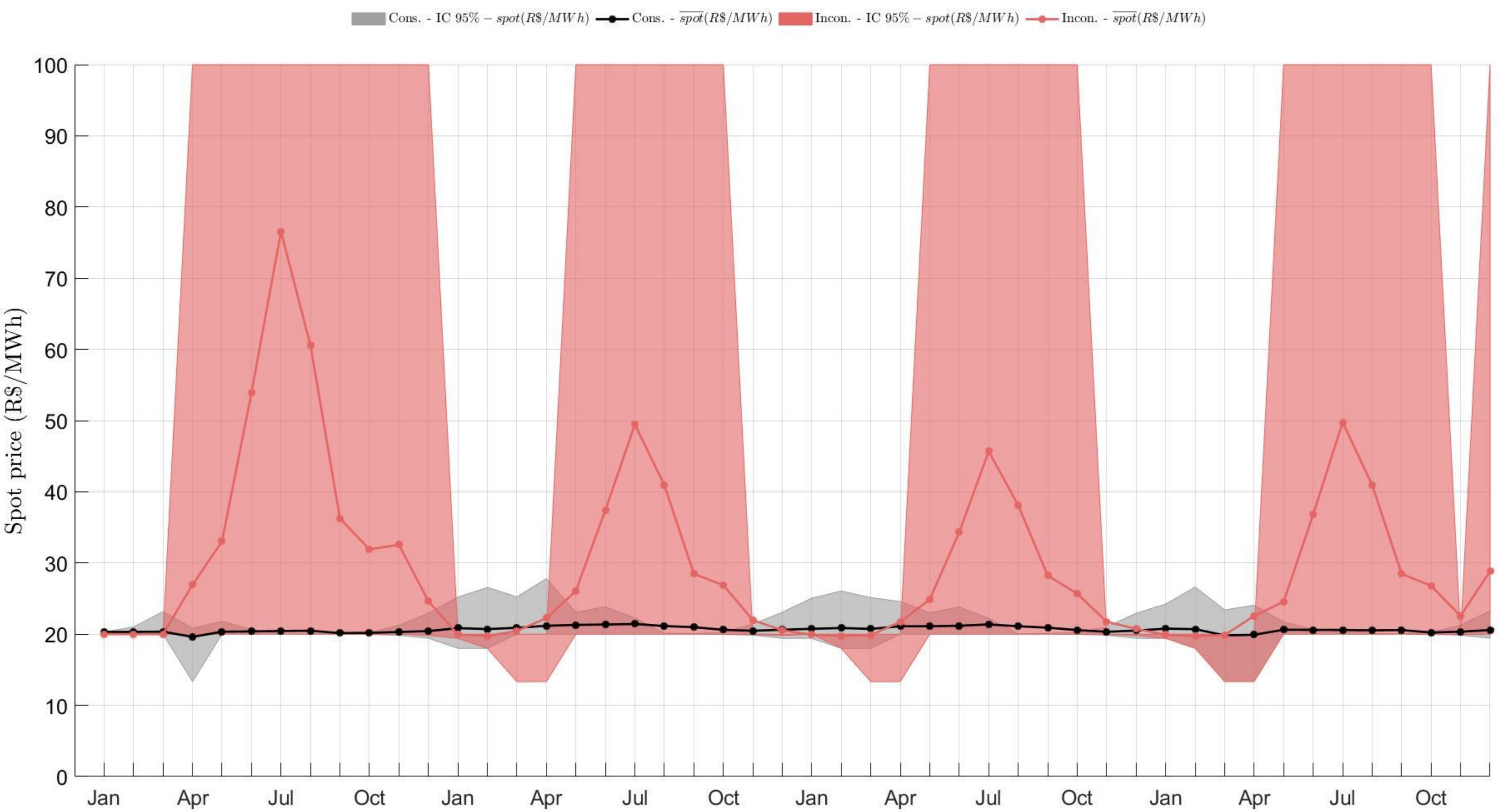
Stored energy



Thermal Generation G_2



Spot price



Cons. - IC 95% – $spot(R\$/MWh)$
Cons. - $\overline{spot}(R\$/MWh)$
Incon. - IC 95% – $spot(R\$/MWh)$
Incon. - $\overline{spot}(R\$/MWh)$
Spot price (R$/MWh)
100
90
80
70
60
50
40
30
20
10
0
Jan
Apr
Jul
Oct
Jan
Apr
Jul
Oct
Jan
Apr
Jul
Oct
Jan
Apr
Jul
Oct

Simplification Example – Security Criteria

$$\min c_t^T g_t + C^{Imb}\delta + \alpha_{t+1}$$

Subject to:

Pre-Contingency

$$A g_t + B y_t + C f_t = d_t$$
$$v_t + u_t + s_t + M(u_t + s_t) = v_{t-1} + w_{t,\omega} : (\tilde{\pi}_{t,\omega}^{(m)})$$
$$f_t = S\theta_t$$
$$\underline{-F} \le f_t \le \overline{F}$$
$$g_t \le \overline{G},\ u_t \le \overline{U}, v_t \le \overline{V}$$

Post-Contingency

$$A^c g_t^c + B^c u_t^c + C^c f_t^c + \phi_t^{c+} - \phi_t^{c-} = d_t; \forall c \in \mathcal{C}$$
$$v_t^c + u_t^c + s_t^c + M(u_t^c + s_t^c) = v_{t-1} + w_{t,\omega} : (\tilde{\pi}_{t,\omega}^{c,(m)}); \forall c \in \mathcal{C}$$
$$f_t^c = S^c \theta_t^c; \forall c \in \mathcal{C}$$
$$v_t^c \le \overline{V}; \forall c \in \mathcal{C}$$
$$-Z_l^c \overline{F}_l \le f_t^c \le Z_l^c \overline{F}_l; \forall c \in \mathcal{C}$$
$$\left\| g_t - g_t^c \right\| \le Z_g^c R_g; \forall c \in \mathcal{C}$$
$$\left\| u_t - u_t^c \right\| \le Z_u^c R_u; \forall c \in \mathcal{C}$$
$$\delta \ge \phi_t^{+c} + \phi_t^{-c}; \forall c \in \mathcal{C}$$

Cut

$$\alpha_{t+1} \ge \tilde{Q}_{t+1}^{(k)}\left(v_t^{(k)}\right) + \left(\tilde{\pi}_{t+1}^{(k)} + \sum_{c \in \mathcal{C}} \tilde{\pi}_{t+1}^{c,(k)}\right)^{\top}\left(v_t - v_t^{(k)}\right); \forall k \in \mathcal{K}^{(m)}$$

Simplification Example – Security Criteria

$$\min c_t^T g_t + C^{Imb}\delta + \alpha_{t+1}$$

Subject to:

χ_t^{plan}

χ_t^{imp}

Pre-Contingency

$$A g_t + B y_t + C f_t = d_t$$
$$v_t + u_t + s_t + M(u_t + s_t) = v_{t-1} + w_{t,\omega} : (\tilde{\pi}_{t,\omega}^{(m)})$$
$$f_t = S\theta_t$$
$$-\overline{F} \leq f_t \leq \overline{F}$$
$$g_t \leq \overline{G}, u_t \leq \overline{U}, v_t \leq \overline{V}$$

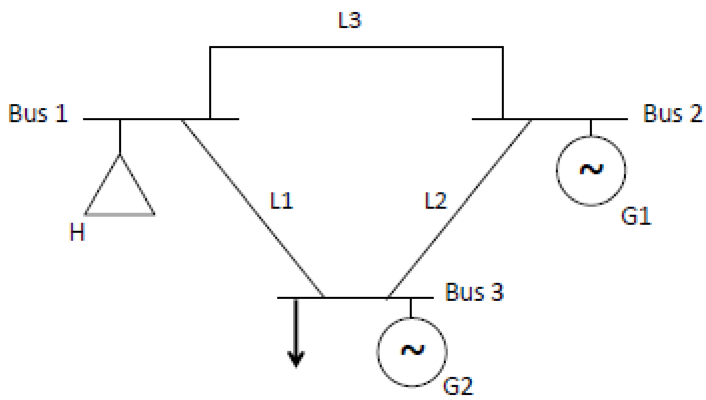
Post-Contingency

$$A^c g_t^c + B^c u_t^c + C^c f_t^c + \phi_t^{c+} - \phi_t^{c-} = d_t; \forall c \in \mathcal{C}$$
$$v_t^c + u_t^c + s_t^c + M(u_t^c + s_t^c) = v_{t-1} + w_{t,\omega} : (\tilde{\pi}_{t,\omega}^{c,(m)}); \forall c \in \mathcal{C}$$
$$f_t^c = S^c \theta_t^c; \forall c \in \mathcal{C}$$
$$v_t^c \leq \overline{V}; \forall c \in \mathcal{C}$$
$$-Z_l^c \overline{F}_l \leq f_t^c \leq Z_l^c \overline{F}_l; \forall c \in \mathcal{C}$$
$$||g_t - g_t^c|| \leq Z_g^c R_g; \forall c \in \mathcal{C}$$
$$||u_t - u_t^c|| \leq Z_u^c R_u; \forall c \in \mathcal{C}$$
$$\delta \geq \phi_t^{+c} + \phi_t^{-c}; \forall c \in \mathcal{C}$$

Cut

$$\alpha_{t+1} \geq \tilde{Q}_{t+1}^{(k)}\left(v_t^{(k)}\right) + \left(\tilde{\pi}_{t+1}^{(k)} + \sum_{c \in \mathcal{C}} \tilde{\pi}_{t+1}^{c,(k)}\right)^{\mathsf{T}} \left(v_t - v_t^{(k)}\right); \forall k \in \mathcal{K}^{(m)}$$

System



- $T = 60$. (Last 12 periods are discarded).
- D = 100MWh.
- **Using $n-1$ in transmission lines only.**

Thermal Generator	c R$/MWh	$\overline{G}$ MW
G_1	20	100
G_2	100	55

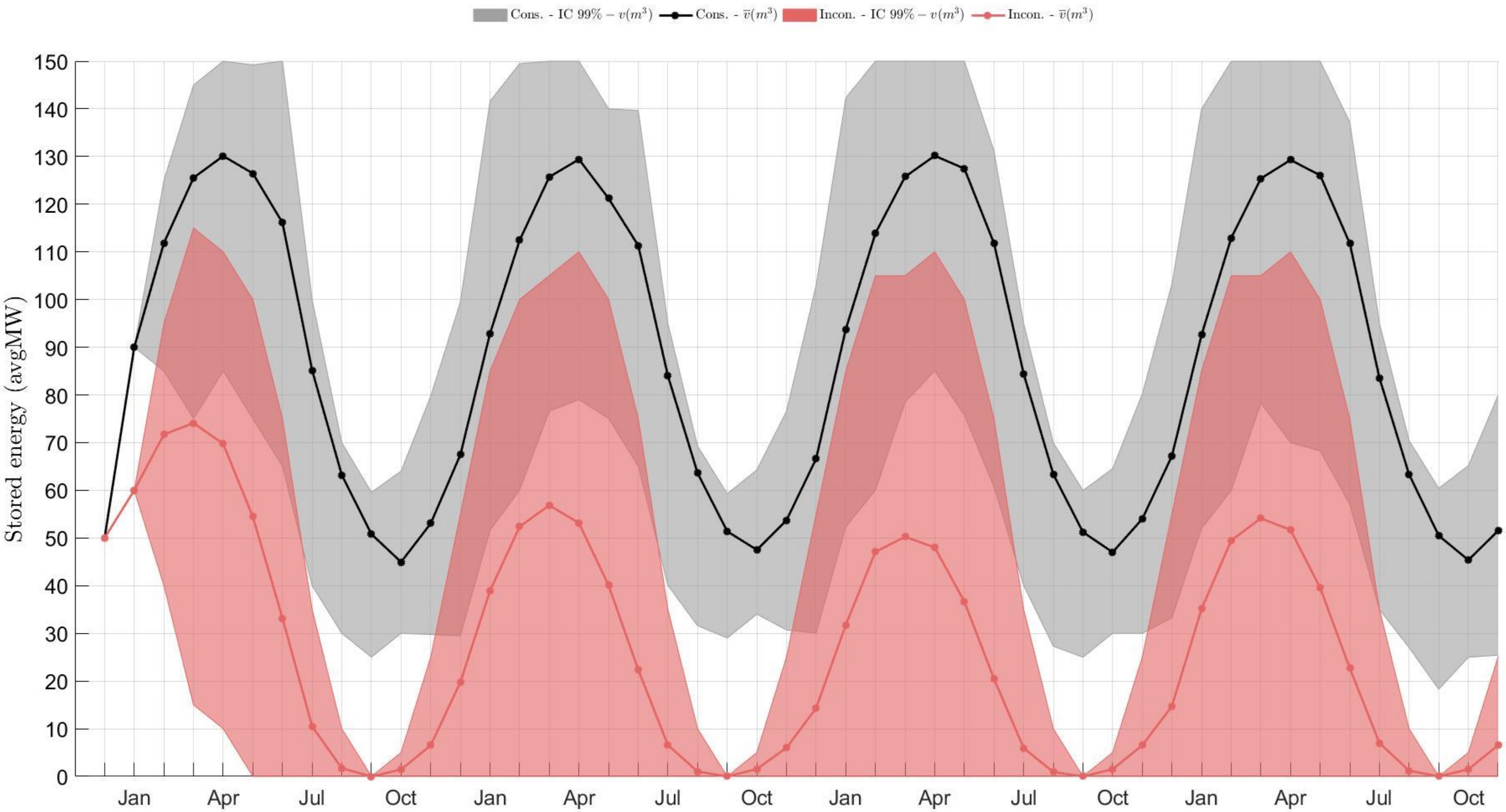
Hydro Generator	$\overline{V}$ m^3	$\overline{U}$ m^3
H	150	80

TL	From	To	$\overline{F}$ MW	Reactance (pu)
1	1	3	100	1
2	2	3	70	1
3	1	2	30	1

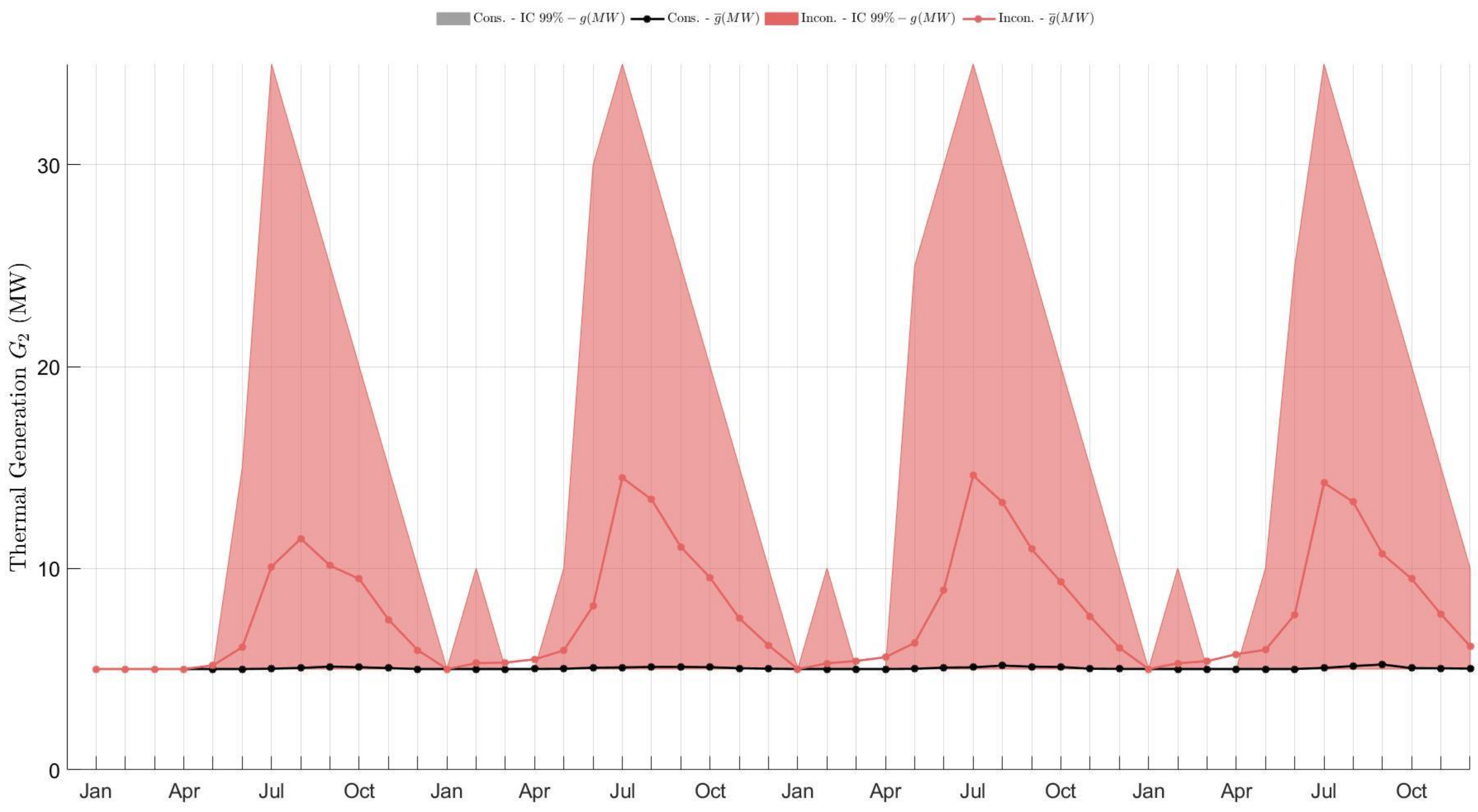
GAP

GAP: MMR$1761.8
GAP confidence interval: MMR$[1737.9 1785.8]

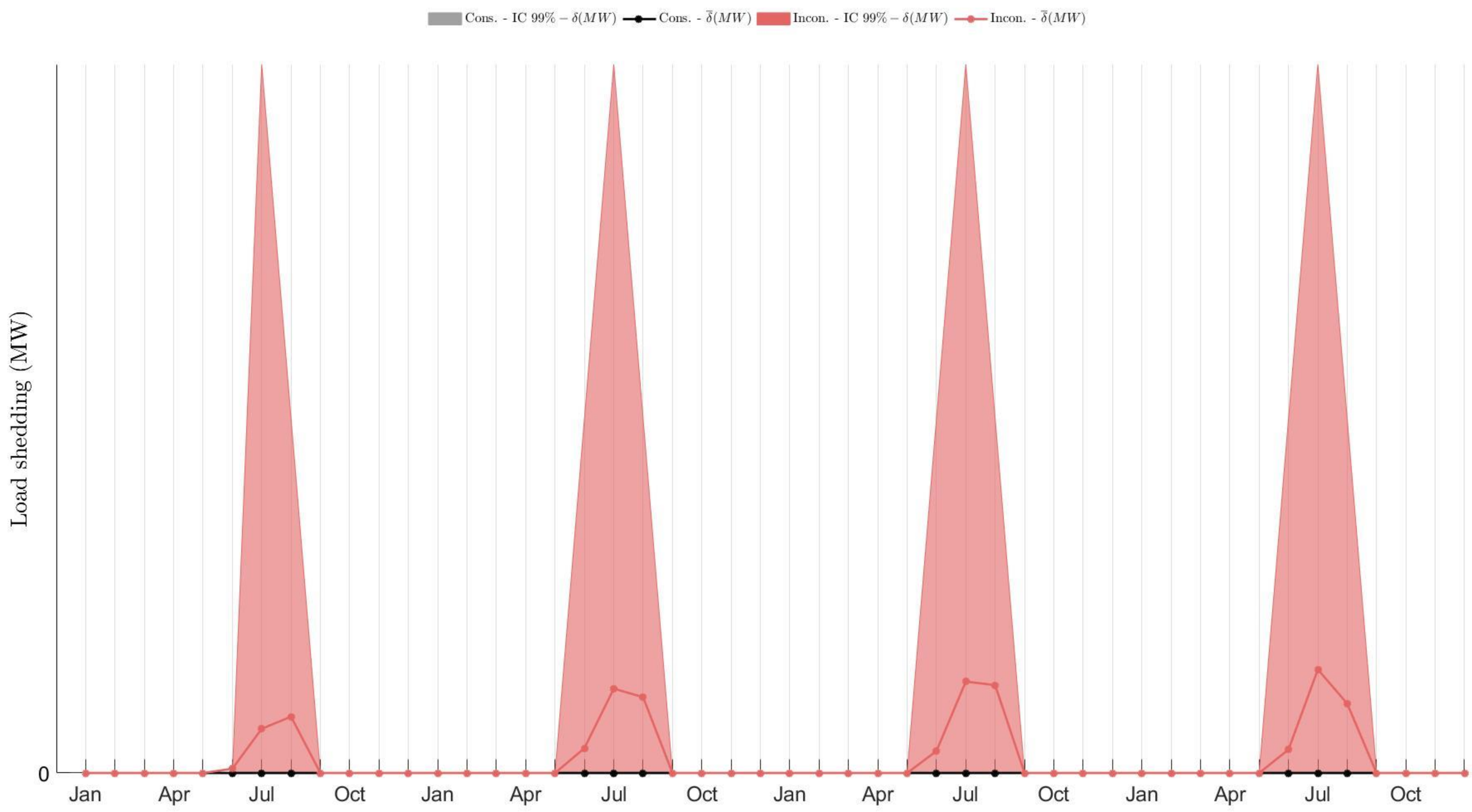
Stored energy



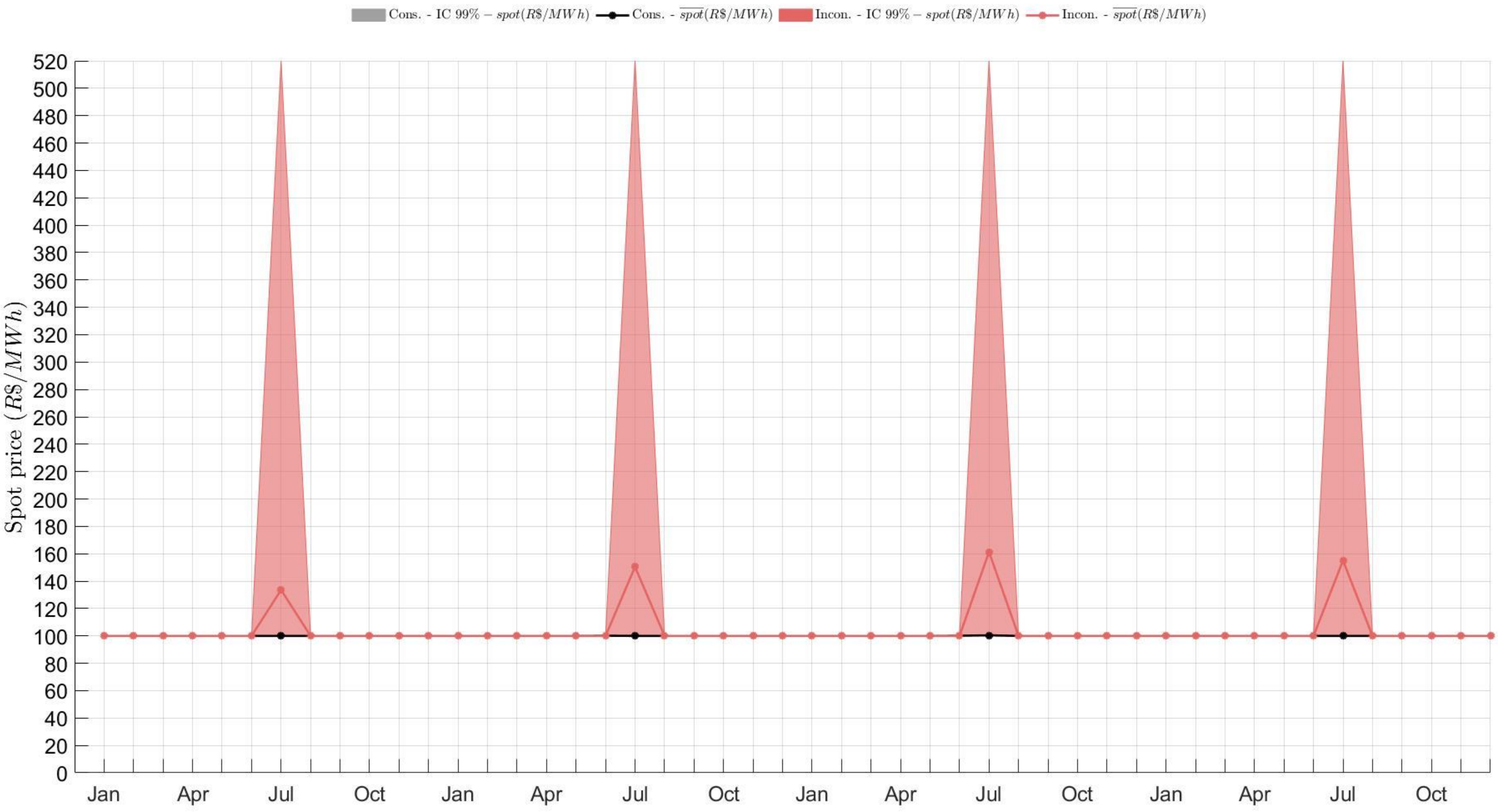
Thermal Generation G_2



Load shedding

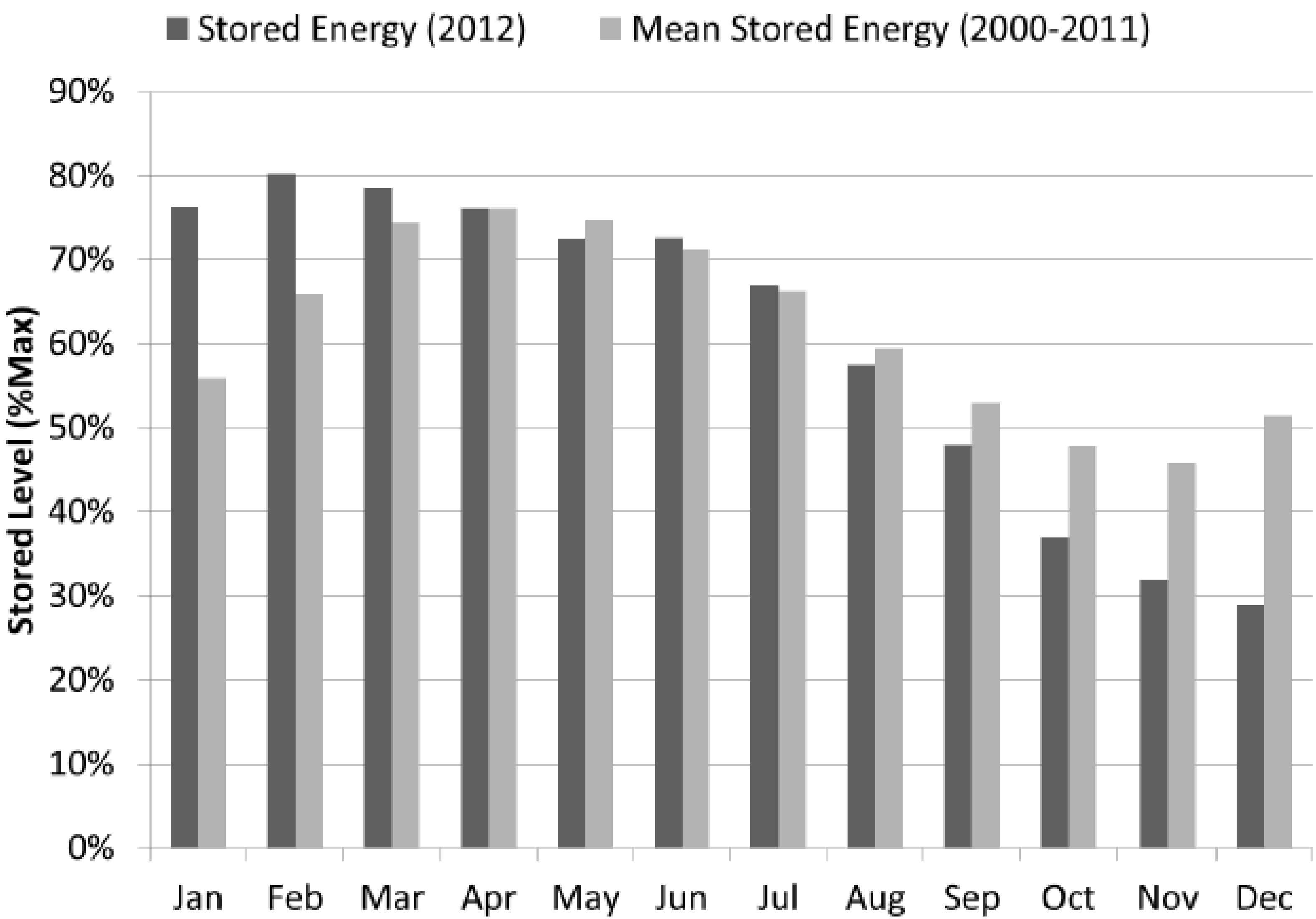


Spot price

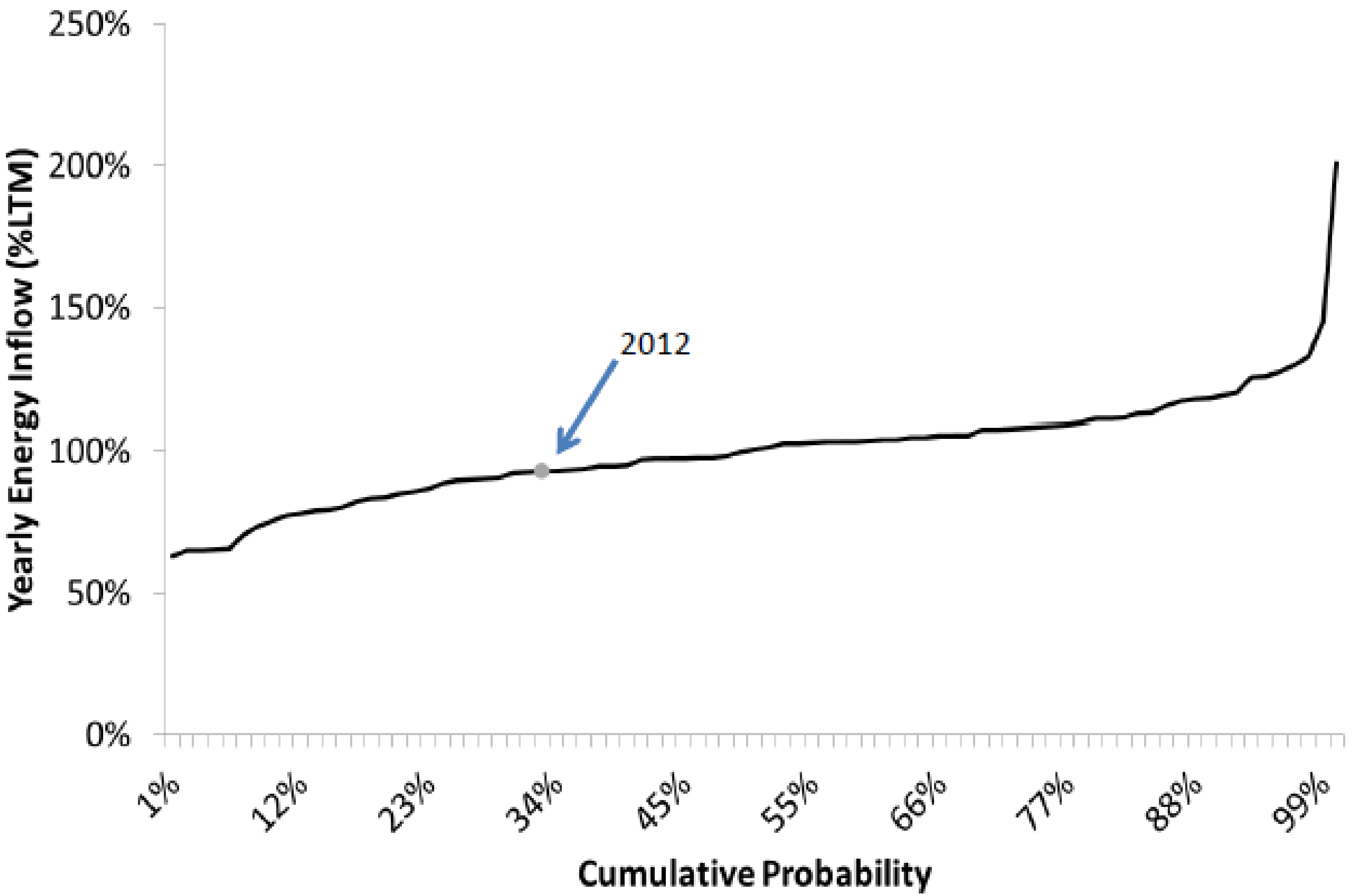


Cons. - IC 99% – $spot(R\$/MWh)$
Cons. - $\overline{spot}(R\$/MWh)$
Incon. - IC 99% – $spot(R\$/MWh)$
Incon. - $\overline{spot}(R\$/MWh)$
Spot price $(R\$/MWh)$
520 500 480 460 440 420 400 380 360 340 320 300 280 260 240 220 200 180 160 140 120 100 80 60 40 20 0
Jan Apr Jul Oct Jan Apr Jul Oct Jan Apr Jul Oct Jan Apr Jul Oct

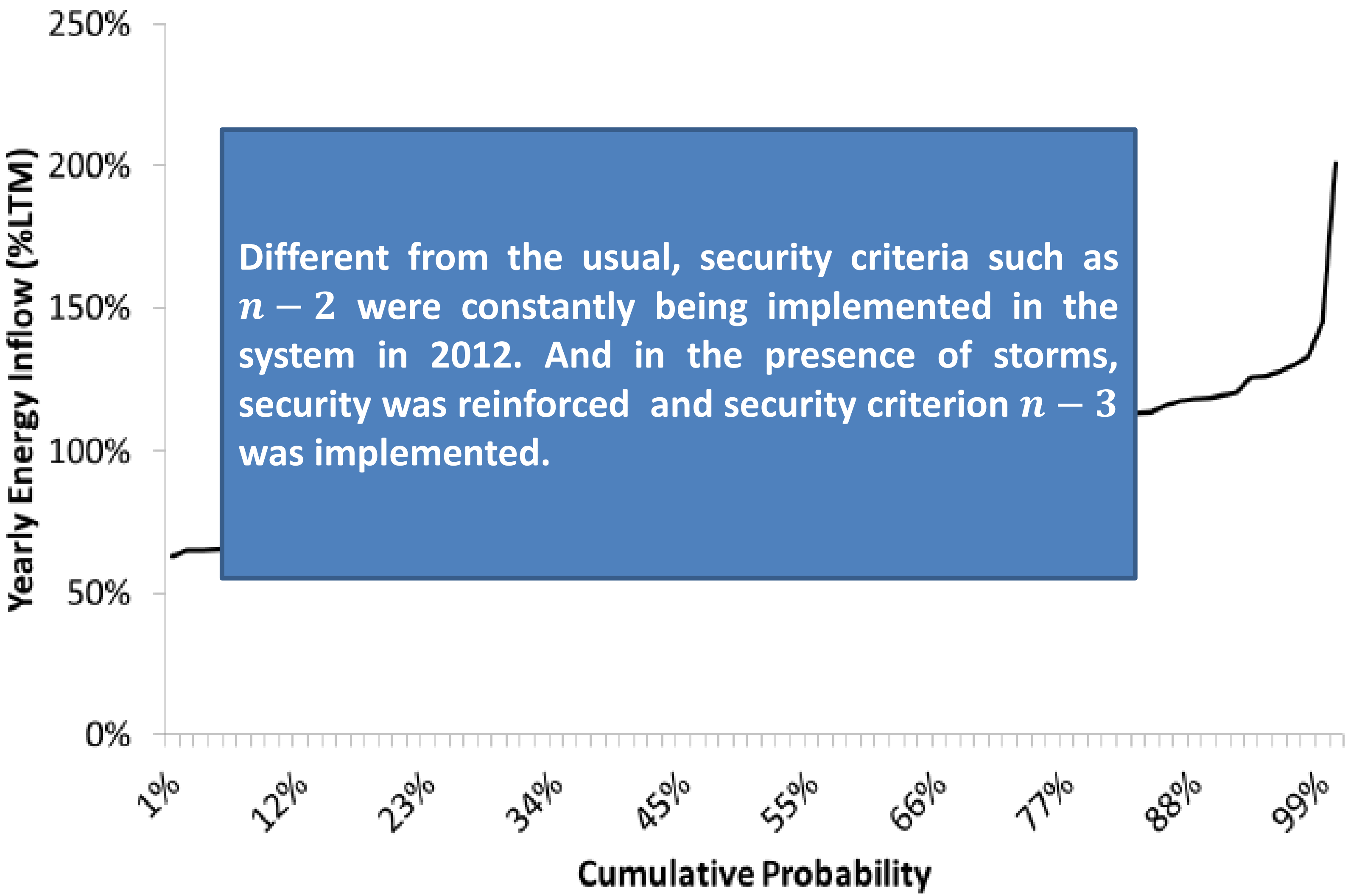
Stored Energy in the Brazilian Southeast Subsystem in 2012



Inflow Energy in the SE Subsystem



Inflow Energy in the SE Subsystem



Different from the usual, security criteria such as $n-2$ were constantly being implemented in the system in 2012. And in the presence of storms, security was reinforced and security criterion $n-3$ was implemented.